Control y Análisis de Estabilidad de un Convertidor DC-DC para Microchips

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MOTIVATION

SYSTEM

DESIGN OF CONTROL LAWS

STABILITY ANALYSIS

DISCRETIZATION AND COMPARISON

CONCLUSIONS
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ARAVIS Project sponsored by Minalogic pole

Currently technology: 90nm, 65nm and, even, 45nm cannot be applied any more to the technology of 32nm.

WHY???

Technology variability phenomenon
Three technology keys:

- Re-configurable structure w.r.t. applicability requirements
- Asynchronous technique.
- Dynamic management of the power consumption and activity
MOTIVATION

STABILITY OF THE CLOSED-LOOP SYSTEM

LDVS
Local Dynamic Voltage Scaling Architecture

GALS
Globally Asynchronous and Locally Synchronous Systems

Vdd-Hopping\(^{(1)}\)

PSS + Two voltage sources

CONTROL !!!!!

Objectives in VLSI:
1. High energy-efficiency.
2. Current peaks limits.
3. Reduction of transient times.
4. Robustness
5. Simple implementation

(1) S. Miermot et al. LECTURE NOTES IN COMPUTER SCIENCE
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Vdd-Hopping DC-DC Converter:

System control model

where \[ R(u_k) = \frac{R}{u} \]
and \[
\begin{align*}
u &= \sum_{i=1}^{n} M_i \\
R &= R_1 = R_2 = \cdots = R_n
\end{align*}
\]
**DC-DC converter circuit**

**Converter model**

\[
\dot{v}_c = -\beta v_c + b(V_h - v_c)u_k - \delta
\]

(1) \[
\begin{cases}
\beta > 0 \\
\delta, b > 0 \text{ and cte}
\end{cases}
\]

**Theorem:** Consider the following:
1. \(v_c, v_r\) and \(u_k\) are such that \(v_c, u_k \in F \subseteq R^+\), for all \(t\), where \(F\) is a bounded subset of positive real numbers, and \(v_r\) is a bounded reference,
2. \(b > 0\) and constant,
3. \(r \leq r_0\), with \(r_0 > 0\),
then System (1) is globally stable in the sense that for all initial condition \(e(0)\), the solutions \(e(t)\) tend to a ball of radius \(r_0\).

**Error equation**

\[
\dot{e} = -(\beta + bu_k)e + \beta v_r + bu_k (v_r - V_h) + \delta
\]

where \(e = v_r - v_c\)
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**DESIGN OF CONTROL LAWS**

**ROBUST CONTROL**

\[ u_k = \text{sat}^N_1 \text{round} \left( K_1 e + K_2 \sigma \right) \]

\[
\begin{align*}
K_1 &= \frac{2\xi \omega_n - (u_{kl}b + \beta_l)}{b(V_{high} - V_{low})} \\
K_2 &= \frac{\omega_n^2}{b(V_{high} - V_{low})}
\end{align*}
\]

being \( \beta_l = \min(\beta) \). Note that \( K_2 > 0 \)

**Lemma 1** If \( \xi \) is chosen such that

\[ \xi \in \left[ \frac{u_{kl}b + \beta_l}{2\omega_n}, \frac{u_{kl}b + \beta_l}{2\omega_n} + \frac{\omega_n^2 b(V_h - V_{low})}{b^2 \bar{u}_k^M(V_h - V_{low})k + 2\omega_n^2} \right] \]

then \( K_1 > 0 \) and \( K_1(b\bar{u}_k^M + 2K_2) - K_2 < 0 \).

Being \( \bar{u}_k^M \) the maximal equilibrium value of \( u_k \).

Note, that \( \frac{\omega_n^2 b(V_h - V_{low})}{b^2 \bar{u}_k^M(V_h - V_{low})k + 2\omega_n^2} > 0 \).
ROBUST CONTROL

\[ u_k = \text{sat}_1^N \, \text{round} \left( K_1 e + K_2 \sigma \right) \]

WITH STEP REFERENCE SIGNAL

BEWARE!!!!!!
This current peak can damage the physical system.
MANAGING CURRENT PEAKS

Maximal current peaks constraint:

\[ 0 < \Delta I \leq |\Delta I_{\text{max}}| \] considering \( v_c \) is continuous

\[ -\Delta I_{\text{max}} \leq \frac{V_{\text{high}} - v_c}{R_0} \Delta u_k \leq \Delta I_{\text{max}} \]

This constraint can be introduced:

\[ u_{k-1} + \Delta u_k^M = u_{k-1} + \frac{R_0}{V_h - v_c} \Delta I_{\text{max}} = u_{k-1} + \alpha_k^M \]

\[ u_{k-1} + \Delta u_k^m = u_{k-1} - \frac{R_0}{V_h - v_c} \Delta I_{\text{max}} = u_{k-1} + \alpha_k^m \]

Thus, the controller will be

\[ u_k = \text{sat}^N \left( \text{round} \left( \text{sat}^{u_{k-1} + \alpha_k^M} \right) \right) \]

\[ R_1 e + K_2 \sigma \]
ROBUST CONTROL WITH CURRENT PEAK CONSTRAINTS

$$u_k = \text{sat}_1^N \left\lfloor \text{round} \left( \frac{\text{sat}_{u_{k-1}}^{\alpha_k^M}}{m} \right) \frac{\text{e}}{K_1} + K_2 \sigma \right\rfloor$$
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Theorem 1: System \( \dot{e} = -(\beta + bu_k)e + \beta v_r + bu_k(v_r - V_h) + \delta \) with the controller \( u_k = \text{sat}^N_1 \) round \( K_1 e + K_2 \sigma \) is locally asymptotically stable for all initial condition \( e(0) \), if \( K_1 \) and \( K_2 \) are positives.

Proof: Let us rewrite error equation

\[
\dot{e} = -f(\bar{u}_k + w_k)e + b(v_r - V_h)(w_k + \bar{u}_k) + K_2 v_c + \delta
\]

where

\[
\begin{cases}
    w_k = u_k - \bar{u}_k \\
    \bar{u}_k = \frac{\beta \bar{v}_c + \delta}{b(V_h - v_r)} = K_2 \bar{\sigma}
\end{cases}
\]

being \( \bar{v}_c, \bar{\sigma} \) equilibrium values.
Candidate Lyapunov function:

\[ V = \frac{e^2}{2b(V_h - v_r)} + \frac{(\sigma - \bar{\sigma})^2}{2} K_2 \]

Derivating and adding \( \pm e^2 \bar{K}_1 \)

\[ \dot{V} = -\left( \frac{f(u_k) + K_2}{b(V_h - v_r)} + K_1 \right) e^2 \leq 0 \quad \text{where } K_1, K_2 \text{ are positives.} \]

The stability is established by LaSalle’s invariant principle, since the maximum invariant set with \( \dot{V} = 0 \) is the single point \( (e = 0, \sigma = \bar{\sigma}) \).
**Theorem 2:** System \( \dot{e} = - (\beta + bu_k)e + \beta v_r + bu_k (v_r - V_h) + \delta \) with the controller \( u_k = sat_1^N \) round \( at^{u_k_{k-1} + a_k^M} \) \( K_1 e + K_2 \sigma \) is locally asymptotically stable for all initial condition \( e(0) \), if \( K_1 \) and \( K_2 \) are positives.

**Proof:**

**Remark:** The equilibriums of the system are in Region I.

\[ \bar{u}_k = sat_1^N \left\{ round \left( sat_{K_2 \bar{\sigma} + \frac{R_0}{V_h - v_c} \Delta I_{\text{max}}} \frac{R_0}{V_h - v_c} \Delta I_{\text{max}} \right) \right\} \]
Lemma 2: System \( \dot{e} = -(\beta + bu_k)e + \beta v_r + bu_k(v_r - V_h) + \delta \) with controller \( u_k = \text{sat}_1^N \text{round}(u_{k-1} + \alpha_k^M) \text{sat}_1^M e + K_2 \bar{\sigma} \) saturated in the upper or lower current peak limit for all initial condition \( e(0) \) converges to the non-saturation region in a finite time, if 
\[ K_1(\bar{u}_k^M + 2\beta(v_c)) - K_2 < 0 \]

Proof: \( \dot{u}_k \rightarrow \alpha_k^M \)

REGION II

Property: In Region II the system fulfills

- \( \dot{u}_k > \alpha_k^M(e) > 0 \)
- \( \ddot{u}_k(e) \leq \varepsilon < 0 \) being \( \varepsilon = -bK_1R_0\Delta I_{\text{max}} = \text{cte} \)
- \( e > 0 \)

Remark: \( \dot{u}_k \) is monotonous decreasing, with derivative bounded away from zero, and hence it will reach the equilibriums in finite time.
Candidate Lyapunov function

\[ W = \dot{u}_k - \alpha_k^M (e) = K_1 \dot{e} + K_2 e - \alpha_k^M (e) > 0 \]

Derivating and substituting the second derivative error equation:

\[ \dot{W} = -b(V_h - v_c)K_1 \dot{u}_k + \left( -K_1 (bu_k + 2K_2) + K_2 + \frac{\alpha_k^M (e)}{V_h - v_r + e} \right) \dot{e} \]

\[ \leq -b(V_h - v_c)K_1 \dot{u}_k + \left( K_1 (bu_k + 2K_2) - K_2 - \frac{\alpha_k^M (e)}{V_h - v_r + e} \right) \frac{K_2 e}{K_1} \]

\[ < 0 \]

\[ \dot{u}_k = K_1 \dot{e} + K_2 e > 0 \]

\[ -\dot{e} < \frac{K_2 e}{K_1} \]

\[ \bar{u}_k > u_k \]

\[ K_1 (b \bar{u}_k^M + 2K_2) - K_2 < 0 \]
By La Salle’s invariance principle, we can conclude the statement of the Theorem, since we have found an invariant set $\Omega$ (Region I), such that $\dot{V}(e, \sigma) = 0 \quad \forall (e, \sigma) \in \Omega$ containing the desired point. Furthermore, Lemma 2 implies that the domain of attraction is not restricted to the invariant set $\Omega$, but it is also limited by the saturation limits and some appropriate Lyapunov levels.

Proof similar to REGION II
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\[ u_k = \text{sat}_1^N \left\{ \text{round} \left( \text{sat}_{u_{k-1}+\alpha_k^m}^u \right) \right\} R_1 e + K_2 \sigma \]

\[ \left\{ \begin{array}{l}
    K_1 = K_1 - \frac{K_2}{2} \\
    K_2 = K_2 T
\end{array} \right. \]

\[ u_k = \text{sat}_1^N \sigma_{k-1}^l + \text{round} \left( \text{sat}_{\alpha_k^m}^u \right) \left( R_1 (e_k - e_{k-1}) + K_2 e_k \right) \]

Control patent pending under the name ENergy-AwaRe Control (ENARC)
COMPARISON

PREVIOUS CONTROL LAW [S. Miermot et al. LNCS]

\[ u_k = u_{k-1} + \text{sign}(e) \]

ENARC

![Graphs showing comparison between previous control law and ENARC](image)
**COMPARISON**

**TOTAL ENERGY DISSIPATED**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Energy (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intuitive controller</td>
<td>7.2 µJ</td>
</tr>
<tr>
<td>ENARC</td>
<td>0.3 µJ</td>
</tr>
</tbody>
</table>

96%
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A control law is proposed for the Vdd-Hopping mechanism in order to:

- Dissipated energy is reduced
- Current peaks are limited
- Transient time are reduced
- Simple implementation
- and robust control.

Closed-loop system stability is analyzed in continuous time.

A discretization of this controller is done.

A comparison with a controller presented in [S. Miermot et al. *LNCS*] is done.
Gracias!!!

Dudas, comentarios??