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Abstract

This paper presents the experimental results of the application of a robust tube-based MPC for tracking of piece-wise constant references to a plant based on the quadruple-tank process. This controller, defined for LTI system subject to additive and bounded disturbances, ensures: (i) the feasibility for any admissible setpoint, (ii) the robust constraint satisfaction, (iii) robust stability and convergence to (a neighborhood of) the desired steady state.

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Robust tubed-based MPC for tracking applied to the quadruple-tank process
I. Alvarado, D. Limon, A. Ferramosca, T. Alamo and E.F. Camacho

Abstract—This paper presents the application of a robust tube-based MPC for tracking of piece-wise constant references to a plant based on the quadruple-tank process [2]. This controller [7], defined for LTI system subject to additive and bounded disturbances, ensures: (i) the feasibility for any admissible setpoint, (ii) the robust constraint satisfaction, (iii) robust stability and convergence to (a neighborhood of) the desired steady state.

I. INTRODUCTION

In this paper, experimental results of the application of the Robust MPC for tracking ([7],[1]) to an experimental tank system developed at the University of Seville are presented. This plant is based on the well known quadruple-tank process [4].

The real plant is assumed to be modeled as a linear system with additive bounded uncertainties on the states. Under mild assumptions, the proposed MPC can steer the uncertain system in an admissible evolution to any admissible steady state, that is, under any change of the set point. This allows us to reject constant disturbances compensating the effect of then, changing the setpoint.

Feasibility of the proposed controller for any admissible setpoint is achieved by adding an artificial steady state as decision variable. Robust constraint satisfaction is guaranteed by tube-based approach and considering nominal predictions. Robust stability and convergence to (a neighborhood of) the desired steady state is ensured by considering a modified cost function and an extended terminal constraint. The cost function penalizes the tracking error with the artificial reference and the deviation between the artificial and desired steady state; the terminal constraint restricts the terminal state and the artificial steady state.

The quadruple tank process is a multivariable laboratory plant of interconnected tanks that can be easily configured to exhibit the effect of multivariable zero (minimum and non-minimum phase) on the system behavior, as well as the effect of non linear dynamics, saturation, constraints, etc.

The real plant has been implemented using industrial instrumentation and a PLC for the low level control. Supervision and control of the plant is carried out in a computer by means of OPC (OLE for Process Control) which allows one to connect the plant with a wide range of control programs such as LabView, Matlab or industrial SCADA.

The paper is organized as follows: in §2 the plant proposed by [4] is presented; the implemented plant is described in the following section. §4 introduces briefly the controller and §5 shows the results of the application of this controller to the plant. The paper draws to a close with some conclusions.

Notation: A positive definite matrix \( T \) is denoted as \( T > 0 \) and \( T > P \) denotes that \( T - P > 0 \). For a given symmetric matrix \( P > 0 \), \( \| x \|_P \) denotes the weighted Euclidean norm of \( x \), i.e. \( \| x \|_P = \sqrt{x^T P x} \). Consider \( a \in \mathbb{R}^{n_a}, b \in \mathbb{R}^{n_b} \), and set \( \Gamma \subset \mathbb{R}^{n_a+n_b} \), then projection operation is defined as \( \text{Proj}_{\Gamma}(a) = \{ a \in \mathbb{R}^{n_a} : \exists b \in \mathbb{R}^{n_b}, \{ a, b \} \in \Gamma \} \). Given two sets \( \mathcal{V} \) and \( \mathcal{Y} \), such that \( \mathcal{V} \subset \mathbb{R}^n \) and \( \mathcal{Y} \subset \mathbb{R}^m \), the Minkowski sum is defined by \( \mathcal{V} \oplus \mathcal{Y} = \{ u+v \mid u \in \mathcal{V}, v \in \mathcal{Y} \} \), the Pontryagin set difference is: \( \mathcal{W} \ominus \mathcal{Y} = \{ u \mid u \ominus \mathcal{Y} \subseteq \mathcal{W} \} \), for a given \( \lambda, \mathcal{X} = \{ \lambda x : x \in \mathcal{X} \} \). Let \( t \) be a generic vector defined as \( t \Delta = \{ t(0), t(1), \ldots \} \).

II. THE QUADRUPLE-TANK PROCESS

In this section the Quadruple-tank process [4] in which the plant is based, is presented (Figure 1). The inputs are the voltages of the two pumps and the outputs are the water levels in the lower two tanks. The parameters of the plant are:

- \( A_i \): Cross-section of tank \( i \).
- \( q_i \): Discharge constant of the tank \( i \).
- \( h_i \): Water level of the tank \( i \) (state of the system).
- \( q_{a}, q_{b} \): Flow produced by the the pumps \( a \) and \( b \).
- \( g \): The acceleration of gravity.
- \( q_i \): Inflow of each tank.
- \( \gamma_i \): Parameters of the three-way valves.
A state space continuous time model of the system can be derived from first principles as follows:

\[
\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a}{A_1}q_a \\
\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b}{A_2}q_b \\
\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{1 - \gamma_a}{A_3}q_b \\
\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{1 - \gamma_b}{A_4}q_a
\]  

(1)

Linearizing the model in an operating point given by \(h_i^0\), \(q_j^0\) and defining the deviation variables \(x_i = h_i - h_i^0\) and \(u_j = q_j - q_j^0\) where \(j = a, b\) and \(i = 1, \cdots, 4\) we have that:

\[
\begin{align*}
\frac{dx}{dt} &= \begin{bmatrix}
-\frac{1}{\tau_1} & 0 & 0 & 0 \\
0 & \frac{1}{\tau_2} & 0 & 0 \\
0 & 0 & \frac{1}{\tau_3} & 0 \\
0 & 0 & 0 & \frac{1}{\tau_4}
\end{bmatrix}x + \begin{bmatrix}
\frac{\gamma_a}{A_1} & 0 & 0 & 0 \\
0 & \frac{\gamma_b}{A_2} & 0 & 0 \\
0 & 0 & \frac{1 - \gamma_a}{A_3} & 0 \\
0 & 0 & 0 & \frac{1 - \gamma_b}{A_4}
\end{bmatrix}u \\
y &= \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}x
\end{align*}
\]  

(2)

where \(\tau_i = \frac{A_i}{\gamma_i^0}\sqrt{\frac{2g}{g}} > 0\), \(i = 1, \cdots, 4\), are the time constants of each tank.

The system is open-loop stable with two multivariable zeros at any given operation point. The nature of these zeros is determined by the parameters \(\gamma_a\) and \(\gamma_b\) as follows:

- If \(0 < \gamma_a + \gamma_b < 1\), the system has right half plane transmission zeros (RHPZ).
- If \(1 < \gamma_a + \gamma_b \leq 2\), the system has left half plane transmission zeros (LHPZ).

It is worth remarking that the sign of the real part of the zeros does not depend on the operating point.

### III. THE REAL PLANT

In the real plant implementation, the original structure of the process has been modified to offer a wide variety of uses for both educational and research purposes. Thus, different plants can be configured such as one single tank, two or three cascaded tanks, a mixture process and hybrid dynamics. Moreover the dynamics parameters of each tank can be set up by tuning the cross-section of the outlet hole of the tank. (See Figure 2)

As it can be seen, this plant is larger that the typical scaled lab plants [13], [5]. Table (I) show the real constraints of the plant.

The tanks have a minimum level under which appears an eddy that changes the value of the effective cross-section of the outlet hole. This implies that the nonlinear model introduced is only valid for states that satisfy these constraints. Table (II) shows the parameters of the plant. Note that this values of \(\gamma_a\) and \(\gamma_b\) have been chosen in order to obtain a system characterized by a non-minimum phase multivariable zeros. Table (III) shows the values of the linearizing equilibrium point.

![Real plant scheme](image)
IV. ROBUST MPC FOR TRACKING

A. Problem description

Consider the following uncertain discrete-time linear time-invariant system:

\[ x^+ = Ax + Bu + w \quad y = Cx + Du \quad (3) \]

where \( x \in \mathcal{X} \subseteq \mathbb{R}^n \), \( u \in \mathcal{U} \subseteq \mathbb{R}^m \) and \( y \in \mathcal{Y} \subseteq \mathbb{R}^p \) is the current state, the current input and the current output of the system; \( x^+ \) is the successor state; \( w \in \mathcal{W} \subseteq \mathbb{R}^n \) is an unknown and bounded state disturbance. \( x(k), u(k), w(k) \) denote the state, the input and the disturbance at sampling time \( k \) of the system (3). \( \mathcal{X}, \mathcal{U}, \mathcal{Y}, \mathcal{W} \) are convex polyhedrons containing the origin in its interior.

**Assumption 1:** System (3) should verify:

- \((A,B,C,D)\) are known and \((A,B)\) controllable.
- The state is accessible.

The proposed controller will be based on the response of the nominal system obtained from (3) by neglecting the disturbances \( w \):

\[ \bar{x}^+ = A\bar{x} + B\bar{u} \quad \bar{y} = C\bar{x} + D\bar{u} \]

\( \bar{x}(i) = \bar{\phi}(i,\bar{x},\bar{u}) \) represent the evolution of the nominal state at time \( i \) if its initial nominal state is \( \bar{x} \) and the control sequence is \( \bar{u} \)

To counteract the disturbances it is desirable to force the trajectory to lie close to the nominal trajectory; this can be done by choosing the control \( u \) to satisfy:

\[ u = \bar{u} + Ke \quad e = (x - \bar{x}) \quad (4) \]

where \( e \) denotes the control error between the state and the state of the nominal system. The error \( e \) satisfies the difference equation:

\[ e^+ = AKe + w \quad AK = (A + KC) \quad (5) \]

**Assumption 2:** The feedback control matrix \( K \) is such that \( AK \) is stable (Hurwitz).

This assumption ensures the existence of a polyhedral robust positively invariant set \( \mathcal{X} \) [6], [12] for the system (5) that satisfies

\[ AK \mathcal{X} \cap \mathcal{W} \subseteq \mathcal{X} \]

From the invariance of \( \mathcal{X} \) the following proposition can be derived:

**Proposition 1:** If the initial real and nominal system states satisfy \( e(0) = x(0) - \bar{x}(0) \in \mathcal{X} \), then \( x(i) \in \bar{x}(i) + \mathcal{X} \quad \forall i \in \mathbb{N} \), and for all admissible disturbance sequences \( w \).

**Theorem 1 ([11]):** Suppose the initial system and nominal system states all lie in \( \mathcal{X} \) and satisfy \( e(0) = x(0) - \bar{x}(0) \in \mathcal{X} \). In addition, if the initial state \( \bar{x}(0) \) and control sequence \( \bar{u} \) of the nominal system satisfy the tighter constraints \( \bar{x}(i) = \bar{\phi}(i,\bar{x},\bar{u}) \in \mathcal{X} \cap \mathcal{W} \) and \( \bar{u}(i) \in \mathcal{U} \cap K\mathcal{X} \) for all \( i \in \mathbb{N} \). Then the state \( x(i) \) and the control \( u(i) = \bar{u}(i) + K(x(i) - \bar{x}(i)) \) of the real system (3) satisfy the original constraints \( x(i) \in \mathcal{X} \) and \( u(i) \in \mathcal{W} \) for all \( i \in \mathbb{N} \) and all admissible disturbance sequences \( w \).

B. Set point characterization

As it was previously presented, the robust tube-based control law allows to use the nominal evolution to predict the uncertain trajectories. Therefore, the set points that can be robustly reached are those steady states of the nominal system admissible considering the following modified set of constraints:

\[ \tilde{\mathcal{X}} = \mathcal{X} \cap \mathcal{W} \quad \tilde{\mathcal{W}} = \mathcal{U} \cap K\mathcal{X} \quad (6) \]

First, following the results presented in [8], the subspace of steady states is parameterized and the admissible set of setpoints characterized. To this aim, consider a given output target \( s \), then any steady state of the nominal system \((x_s, u_s)\) associated to this output must satisfy the following equation

\[ \begin{bmatrix} A - I_n & B & 0_{n,1} \\ C & D & -I_p \end{bmatrix} \begin{bmatrix} x_s \\ u_s \\ s \end{bmatrix} = \begin{bmatrix} 0_{n,1} \\ 0_{p,1} \end{bmatrix} \quad (7) \]

Because the pair \((A,B)\) is stabilizable, the solution to this problem can be parameterized as [8]

\[ \begin{bmatrix} x_s \\ u_s \\ s \end{bmatrix} = M_\theta \theta \quad s = N_\theta \theta \quad (8) \]

where \( \theta \in \mathbb{R}^{n_{\theta}} \) is a parameter vector which characterizes any solution, and \( M_\theta \) and \( N_\theta \) are suitable matrices. The existence of constraints (6) limit the set of reachable steady states and inputs. The set of admissible steady states is denoted as \( \mathcal{X}_s \) and it is a polyhedron given by

\[ \mathcal{X}_s = \{ x_s \in \mathcal{X} : \exists u_s \in \mathcal{U} \quad |(A - I_n)x_s + Bu_s = 0_{n,1}\} \]

The set of admissible steady outputs is denoted as \( \mathcal{Y} \) and it is a polyhedron given by

\[ \mathcal{Y} = \{ s \in \mathcal{Y} : x_s \in \mathcal{X}_s, u_s \in \mathcal{U} \quad |(A - I_n)x_s + Bu_s = 0_{n,1} \quad and \quad s = Cx_s + Du_s\} \]

C. Plant Operation Point to be tracked

The setpoint can be provided as a output target \( s \), an equilibrium state \( x_s \) or input \( u_s \) of the linear model or an target state that it is not an equilibrium point of the linear model \( x_t \). The most compact way to characterize an equilibrium point for the linear model is the parameter \( \theta \); thus, the operating point provided in this first three cases can be described by a value of \( \theta \) that can be easily calculated from (8). Therefore, the proposed control law will depend on the current state and the parameter \( \theta \), that is, \( u = \kappa_\theta(x, \theta) \).

In the case that the operating point is not an equilibrium point of the linear model there is an alternative formulation of the controller that can be found in [1].

It is worth remarking that the admissibility of the operating point to be tracked is not an issue on this stage because the proposed predictive controller ensures the admissible trajectory of the system despite the operating point.

D. Invariant set for tracking

The invariant set for tracking is the set of initial points that due to a linear stabilizing control law will converge to one admissible equilibrium point in an admissible way

**Definition 1:** Let \( x^0 \) be the extended state \((x, \theta) \in \mathbb{R}^{n+n_{\theta}} \), let \( K_\Omega \) be a control gain such that \( A + BK_\Omega \) is Hurwitz and let
$K_0$ be given by $K_0 = [-K_\Omega I_n]M_\theta$. Then, a set $\Omega_\theta' \subset \mathbb{R}^{n+n_\theta}$ is an invariant set for tracking, if for all $(x, \theta) \in \Omega_\theta'$, then $((A + BK_\theta)x + BK_\theta \theta, \theta) \in \Omega_\theta'$.

See that for any $(x(0), \theta) \in \Omega_\theta'$, the trajectory of the system $x(i+1) = Ax(i) + Bu(i)$ controlled by $u(i) = K_\theta(x(i) - x_\theta) + u_\theta$, where $(x_\theta, u_\theta) = M_\theta \theta$, is confined in $\Omega = \text{Pro}j_z(\Omega_\theta')$ and tends to $x_\theta$. Moreover this trajectory is admissible if $\Omega_\theta' \subset \mathbb{R}^n \times \mathbb{R}^n_\theta$ is an invariant set for tracking, if for all $(x, \theta) \in \Omega_\theta'$, $t \in \mathbb{R}^n_\theta$ is an invariant set for tracking, if for all $(x, \theta) \in \Omega_\theta'$, $t \in \mathbb{R}^n_\theta$ is an invariant set for tracking, if for all $(x, \theta) \in \Omega_\theta'$.

$\mathbf{E. Controller formulation}$

In order to ensure the feasibility of the problem for any desired steady state $(x_\theta, u_\theta) = M_\theta \theta$, an artificial steady state $(\bar{x}_\theta, \bar{u}_\theta) = M_\theta \bar{\theta}$ is introduced as a decision variable in the minimization of the performance index. Moreover, robust convergence to the desired steady state is ensured by adding a term $\| \theta - \bar{\theta} \|^2_T$ in the cost function (offset cost) that penalizes the deviation between the desired steady state and the artificial one (see that any quadratic term weighting the offset, as $\| \bar{x}_\theta - x_\theta \|^2_S$ for instance, can be expressed in this form), the initial state of the nominal system $\bar{x}$ is also a decision variable that have to be inside of $x \oplus \mathcal{Z}$. Thus if $x - \bar{x} = e(0) \in \mathcal{Z}$, then $e(i) \in \mathcal{Z}$, $\forall i \in \mathbb{N}$.

Let the cost be defined by:

$$V_N(x, \theta; \bar{u}, \bar{x}, \bar{\theta}) = \sum_{k=0}^{N-1} \| \bar{x}(i) - \bar{x}_\theta \|^2_Q + \| \bar{u}(i) - \bar{u}_\theta \|^2_R + \| \bar{x}(N) - \bar{x}_\theta \|^2 + \| \bar{\theta} - \bar{\theta} \|^2_T$$

(9) This cost penalizes the deviation between the predicted trajectory and the artificial steady state along the horizon $N$ and the deviation between the artificial and the desired steady state.

The formulation of the optimal problem $\mathcal{P}_N(x, \theta)$ is:

$$\min_{u, x, \theta} V_N(x, \theta; \bar{u}, \bar{x}, \bar{\theta})$$

s.t. $\bar{x} \in x \oplus ( - \mathcal{Z})$

$$\bar{x}(i) \in \mathcal{Z} = \mathcal{X} \oplus \mathcal{Z}$$

$$\bar{u}(i) \in \mathcal{U} = \mathcal{U} \oplus K \mathcal{Z}$$

$$\langle \bar{x}(N), \bar{\theta} \rangle \in \Omega_\theta'$$

where $\Omega_\theta'$ is an invariant set for tracking associated to the linear control law described in (IV-D) with a gain $K_\theta$ that can be different to $K$ (5).

Due to the constraints set does not depend on $x_\theta$, the optimization problem $\mathcal{P}_N(x, \theta)$ has a feasible solution $\forall x \in \mathcal{Z}_N \subset \mathbb{R}^2$. The optimal solution of $\mathcal{P}_N(x, \theta)$ is denoted by $\ast$, thus $V_N(x, \theta; \bar{u}, \bar{x}, \bar{\theta})$ denotes the optimal cost , $\bar{u}^*(x, \theta)$, $\bar{x}^*(x, \theta)$ and $\theta^*(x, \theta)$ denote the optimal value of the decision variables, $\bar{x}^*(x, \theta)$ denotes the nominal optimal trajectory and $(\bar{x}^*(x, \theta), \bar{u}^*(x, \theta))$ denote the optimal artificial reference.

The control action is calculated from the optimal solution as follows

$$K_N(x, \theta) = K(x - \bar{x}^*(x, \theta)) + \bar{u}^*(0; x, \theta)$$

(11) where $\bar{u}^*(0; x, \theta)$ is the first component of $\bar{u}^*(x, \theta)$.

It is important to remark that in the case that the disturbances set, the constraints set, the robust invariant set $\mathcal{Z}$ and the invariant set $\Omega_\theta'$ are polyhedra, the proposed problem can be formulated as a QP (Quadratic Programming Problem) that can be solved using efficient algorithms [3], [10].

A more detailed description of this controller can be found in [7].

$\mathbf{V. APPLICATION OF THE ROBUST MPC FOR TRACKING TO THE PLANT}$

$\mathbf{A. Model of the plant}$

The proposed controller requires a LTI model subject to additive and bounded disturbances (3). The value of the matrices are the ones of the linearized model (2) corresponding to the values of the tables (II) and (III) for a sample time $T_m = 5$ sec

$$A = \begin{bmatrix} 0.9701 & 0 & 0.0211 & 0 \\ 0 & 0.9673 & 0 & 0.0192 \\ 0 & 0 & 0.9786 & 0 \\ 0 & 0 & 0 & 0.9804 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0068 & 0.0001 \\ 0.0002 & 0.0091 \\ 0 & 0.0137 \\ 0.0160 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The set $\mathcal{W}$ identified from the experimental data is defined by the following inequality:

$$\| w \|_\infty \leq 5 \times 10^{-3}$$

In this case the setpoint will be provided as an output reference $s$. The calculation of $\theta$ from $s$ in this case is quite simple due to the expression of the matrix $N_\theta$ (8) is:

$$N_\theta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\mathbf{B. Controller design}$

The controller has been designed following the method proposed in [9].

The following stage cost matrices have been chosen. These matrices define the performance criterion that the MPC controller optimizes.

$$Q = C' \times C \quad R = 1 \times 10^{-4} \times I_4$$

(12) $K$ plays an important role in the proposed controller since this control gain is used to compensate the deviation from the nominal predictions in the controller and therefore, it
characterizes the dynamics of the closed loop system in the presence of disturbances. We have used the design method proposed in [9, Section 4.3] for a $\rho = 1$. The resulting controller gain is:

$$ K = \begin{bmatrix} -5.9997 & -18.7429 & 6.2544 & -37.0666 \\ -20.6413 & -12.8487 & -29.7042 & -3.0337 \end{bmatrix} $$

The parameter $K_\Omega$ has been determined by means of a LQR that stabilizes the uncertain system in an admissible way with the same stage cost parameters as before (12). The controller gain is:


The matrix that penalizes the difference between the artificial steady state and the desired one is

$$ T = 1 \times 10^4 \times \begin{bmatrix} 1.3107 & 1.0743 & 1.4670 & 0.7893 \\ 1.0743 & 1.7371 & 0.5064 & 2.4905 \\ 1.4670 & 0.5064 & 2.5372 & -0.8392 \\ 0.7893 & 2.4905 & -0.8392 & 4.9117 \end{bmatrix} $$

The control horizon has a value $N = 3$.

Figure 3 shows:

- the sets $C\mathcal{Z}$ and $C\mathcal{Z}^r$ that are the projection over the outputs $h_1$ and $h_2$ of the minimal robust invariant set $\mathcal{Z}$ and the state constraints set $\mathcal{Z}^r$.
- the sets $\mathcal{W}$ that is the control constraints set and $K\mathcal{Z}$.
- the set of admissible setpoints $\mathcal{S}$ and $C\Omega^r_t$ the projection over the outputs $h_1$ and $h_2$ of the invariant set for tracking $\Omega^r_t$,
- and the control constraints set for the nominal system $\mathcal{W}^\Delta = \mathcal{W} \ominus K\mathcal{Z}$.

Figure 4 shows output trajectories of the simulation experiment (solid line), the set of the admissible setpoints $\mathcal{S}$ and the references (circles). The top part of the Figure 5 shows the time trajectories of the output (solid lines) and the references (dashed lines). The middle part shows the time evolution of the control actions and the bottom part the trajectory of the levels of the tanks 3 and 4.

Figure 5 shows the typical step response of systems that have right half plane transmissions zeros. It can be seen how the levels react in an opposite direction to all reference step changes. Nevertheless, the proposed controller steer the system to the setpoint in an admissible way.

D. Experimental results

Once the operation of the controller is verified by simulation, it is applied to a real system.

The same test is performed on the real plant in order to compare the results by simulation with the real ones. Figure 6 shows the real evolution of the outputs, the set of the admissible setpoints $\mathcal{S}$ and the references. Notice that there is an offset, due to the persistent nature of the disturbances, nevertheless, the evolution are quite similar.

Figure 7 presents the same distribution as Figure 5. In order to achieve an offset free output tracking, a modified setpoint $s$ is provided to the controller $s(k) = s_d - (C + DK)(I_n - (A + BK))^T\hat{w}(k)$ ($s_d$ is the desired setpoint), thus, the offset will compensated if the disturbances tends to a constant value and

![Fig. 3. Different sets of the MPC for tracking applied to the quadruple tank process](image-url)

![Fig. 4. Simulation of the evolution of the outputs](image-url)

![Fig. 5. Simulation of the time evolution of the plant](image-url)
using the appropriate observer [7]. To this end, the modified setpoint must be allowable, which is ensured if the rank of matrix $E$ is $n + p$.

$$E = \begin{bmatrix} A - I_n & B \\ C & D \end{bmatrix}$$

The following estimator is used:

$$\hat{w}_k = \lambda_f \hat{w}_{k-1} + (1 - \lambda_f) (\hat{x}_k - A \hat{x}_{k-1} - B u_{k-1}), \quad \lambda_f = 0.98$$

Figures 8 and 9 show the time evolution of the plant when the corrected setpoint is provided.

VI. CONCLUSIONS

In this paper we have been presented the application of the Robust MPC for Tracking to a a nonlinear uncertain multivariable process configured to work at an operation point characterized by non-minimum phase multivariable zeros. This plant that has been implemented using industrial instrumentation.

This controller ensures the feasibility, the stability and the convergence to a neighborhood of the desired setpoint.

REFERENCES