Adaptive rejection of unknown disturbances

Application to active vibration control

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Sevilla
October 2004

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Outline

- Introduction
- Rejection of unknown stationary disturbances. *Basic facts*
- Rejection of unknown narrow band disturbances in active vibration control. *Real-time results*
- Adaptive control solutions (indirect/direct)
- Further experimental results
  (*comparison direct/indirect adaptive control*)
- Conclusions
Unknown disturbance rejection – classical solution

Disadvantages:

- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters
Rejection of unknown disturbances

- **Assumption:** Plant model almost constant and known (obtained by system identification)
- **Problem:** Attenuation of unknown and/or variable stationary disturbances *without using an additional measurement*
- **Solution:** Adaptive feedback control
  - Estimate the model of the disturbance
  - Use the internal model principle
  - Use of the Youla parameterization (direct adaptive control)

**A class of applications:** suppression of unknown vibrations
*(active vibration control)*

**Attention:**
The area is “dominated “ by adaptive signal processing solutions
*(Widrow’s adaptive noise cancellation)* which require an additional transducer

**Most surprising:** there is an elegant “direct adaptive control” solution

**Remainder:** Models of stationary disturbances have poles on the unit circle
Internal Model Principle

For asymptotic rejection of a disturbance the controller should incorporate the model for the disturbance

\[ \delta \quad \text{Disturbance Model} \quad \text{Disturbance} \]

Remember:

- Step disturbance model: \[ \frac{1}{s} \text{ or } \frac{1}{1-q^{-1}} \]

The controller should incorporate an « integrator » (which is the model of the disturbance)
Indirect adaptive control

Two-steps methodology:

1. Estimation of the disturbance model, $D_p(q^{-1})$

2. Compute the controller using the « internal model principle » (the controller contains the model of the disturbance)

It can be time consuming (if the plant model $B/A$ is of large dimension)

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Direct adaptive rejection of unknown disturbances

- One directly adapts a filter which is part of the controller.
- It tunes the « internal model » inside the controller
- Does not changes the poles of the closed loop (Y-K param.)

Model = Plant
\[ w = Ap \]
Direct adaptive rejection of unknown disturbances

Equivalent representation of the scheme (case $A = \text{as.stable}$)
Rejection of unknown narrow band disturbances in active vibration control
The Active Suspension System

**Objective:**
- Reject the effect of unknown and variable narrow band disturbances
- Do not use an additional measurement

Two paths:
- Primary
- Secondary (double differentiator)

\[ T_s = 1.25 \text{ ms} \]
The Active Suspension

Active suspension
Residual force (acceleration) measurement
Primary force (acceleration) (the shaker)

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Active Suspension

Frequency Characteristics of the Identified Models

Primary path

Secondary path

\[ n_A = 14 \ ; \ n_B = 16 \ ; \ d = 0 \]

Further details can be obtained from: http://iawww.epfl.ch/News/EJC_Benchmark/

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Direct Adaptive Control: disturbance rejection

Disturbance: Chirp

Open loop

Closed loop

Initialization of the adaptive controller

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Time Domain Results

Adaptive Operation

Direct adaptive control

Commande adaptative directe en adaptatif
Initialization of the adaptive controller

Direct Adaptive Control

Output

Input

Adaptation transient

Adaptation transient

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Notations

\begin{align*}
G(q^{-1}) &= \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \\
K(q^{-1}) &= \frac{R(q^{-1})}{S(q^{-1})} = \frac{R'(q^{-1})H_S(q^{-1})}{S'(q^{-1})H_R(q^{-1})}
\end{align*}

Output Sensitivity function:

\[ S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})} \]

Closed loop poles:

\[ P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1}) \]

The gain of \( S_{yp} \) is zero at the frequencies where \( S_{yp}(e^{j\omega}) = 0 \) (perfect attenuation of a disturbance at this frequency)
**Disturbance model**

**Deterministic framework**

\[ p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance} \]

\[ D_p \rightarrow \text{poles on the unit circle}; d(t) = \text{Dirac} \]

**Stochastic framework**

\[ p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t) : \text{stochastic disturbance} \]

\[ D_p \rightarrow \text{poles on the unit circle}; e(t) = \text{Gaussian white noise sequence } (0, \sigma) \]
Closed loop system. Notations

\[ p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance} \]

\[ D_p \rightarrow \text{poles on the unit circle; } d(t) = \text{Dirac} \]

Controller:

\[ R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1}); \]
\[ S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}). \]

Internal model principle: \( H_S(z^{-1}) = D_p(z^{-1}) \)

Output: \( y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \)
\[ y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot \delta(t) \]

CL poles: \( P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d} B(q^{-1})R(q^{-1}) \)

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Indirect adaptive control

Two-steps methodology:

1. Estimation of the disturbance model, \( D_p(q^{-1}) \)

2. Computation of the controller, considering \( H_s(q^{-1}) = \hat{D}_p(q^{-1}) \)

It can be time consuming (if the plant model B/A is of large dimension)

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Indirect adaptive control

*Step I:* Estimation of the disturbance model
ARMA identification (Recursive Extended Least Squares)

*Step II:* Computation of the controller
Solving Bezout equation (for \( S' \) and \( R \))

\[
H_s = \hat{D}_p
\]

\[
A\hat{D}_p S' + q^{-d} BR = P
\]

\[
S = D_p S'
\]

*Remark:*
*It is time consuming for large dimension of the plant model*
Central contr: \([R_0(q^{-1}), S_0(q^{-1})]\).

CL poles: \(P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^d B(q^{-1}) R_0(q^{-1})\).

Control: \(S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)\)

Q-parameterization: 
\[
\begin{align*}
R(z^1) &= R_0(q^{-1}) + A(q^{-1}) Q(q^{-1}) \\
S(z^{-1}) &= S_0(z^{-1}) - q^d B(q^{-1}) Q(q^{-1})
\end{align*}
\]

Control: \(S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t) - Q(q^{-1}) w(t)\),

where \(w(t) = A(q^{-1}) y(t) - q^d B(q^{-1}) u(t)\).

CL poles: \(P(q^{-1}) = A(q^{-1}) S_0(q^{-1}) + q^d B(q^{-1}) R_0(q^{-1})\).
**Internal model principle and Q-parameterization**

Central contr: \([R_0(q^{-1}), S_0(q^{-1})]\).

CL Poles: \(P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^d B(q^{-1})R_0(q^{-1}).\)

Control: \(S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)\)

Q-parameterization:

- \(R(z^1) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1});\)
- \(S(q^{-1}) = S_0(z^1) - q^d B(q^{-1})Q(q^{-1}).\)

Closed Loop Poles remain unchanged

**Internal model assignment on Q**

\[S = S_0 - q^{-d} BQ = MD_p\]

Solve: \(MD_p + q^{-d} BQ = S_0\)

Q \((q^{-1})\) computed such as \([S(q^{-1})]\) contains the internal model of the disturbance

Will lead also to an "indirect adaptive control solution"

**BUT:**

Q can be used to “directly” tune the internal model without changing the closed loop poles (see next)
**Direct Adaptive Control (unknown D<sub>p</sub>)**

(Based on an idea of Y. Z. Tsypkin)

**Hypothesis:** Identified (known) plant model (A,B,d).

**Goal:** minimize y(t) (according to a certain criterion).

Consider \( p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) \): deterministic disturbance.

\[
y(t) = \frac{A(q^{-1})}{P(q^{-1})} \cdot q^{-d}B(q^{-1})Q(q^{-1}) \cdot N_p(q^{-1}) \cdot \delta(t) = \left[ S_0(q^{-1}) - q^{-d}B(q^{-1})Q(q^{-1}) \right] w(t)
\]

Define:

\[
\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d}B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t).
\]

Let \( \hat{Q}(t,q^{-1}) \) be an estimated value of \( Q(q^{-1}) \)

Leads to a direct adaptive control

\[
\varepsilon(t + 1) = [Q(q^{-1}) - \hat{Q}(t + 1, q^{-1})] \cdot \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t + 1)
\]

\( (v(t + 1) = \text{disturbance term} \rightarrow 0) \)

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The Algorithm

**A priori** adaptation error:
\[ \varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t); \]

**A posteriori** adaptation error:
\[ \varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t), \]

where
\[ \hat{\theta}^T(t) = [\hat{q}_0(t) \hat{q}_1(t)]; \quad \phi^T(t) = [w_2(t) w_2(t-1)] \quad \text{(for } n_Q = 1) \]

and
\[ w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1); \]
\[ w_2(t) = \frac{q^{-d}B^*(q^{-1})}{P(q^{-1})} \cdot w(t); \]
\[ w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t); \]
\[ B(q^{-1}) \cdot u(t+1) = B^*(q^{-1}) \cdot u(t). \]

Parameter adaptation algorithm:
\[
\begin{cases}
\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\phi(t)\varepsilon^0(t+1); \\
F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t).
\end{cases}
\]
Direct adaptive rejection of unknown disturbances

- The order of the $Q$ polynomial depends upon the order of the disturbance model denominator ($D_p$) and not upon the complexity of the plant model.
- Less parameters to estimate than for the identification of the disturbance model.
Further experimental results on the active suspension

*Comparison between direct/indirect adaptive control*
Real-time results

Narrow band disturbances = variable frequency sinusoid \( \Rightarrow n_Q = 1 \)
Frequency range: 25 \( \div \) 47 Hz

Evaluation of the two algorithms in real-time

Nominal controller \([R_0(q^{-1}),S_0(q^{-1})]\): \(n_{R_0} = 14\), \(n_{S_0} = 16\)

Implementation protocol 1: Self-tuning

- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample
Frequency domain results – indirect adaptive method

Spectral densities of the residual force. Indirect method in self-tuning operation

- Open loop (25 Hz)
- Open loop (32 Hz)
- Open loop (47 Hz)
- Closed loop (25 Hz)
- Closed loop (32 Hz)
- Closed loop (47 Hz)
Frequency domain results – direct adaptive method

Spectral densities of the residual force. Direct method in self-tuning operation

- Open loop (25 Hz)
- Open loop (32 Hz)
- Open loop (47 Hz)
- Closed loop (25 Hz)
- Closed loop (32 Hz)
- Closed loop (47 Hz)

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**Time Domain Results**

*Self-tuning Operation*

**Indirect adaptive method**

**Direct adaptive method**

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• **Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control**
• **Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode**
Conclusions

- Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances.
- Both direct and indirect solutions can be provided.
- Two modes of operation can be used: self-tuning and adaptive.
- Direct adaptive control is the simplest to implement.
- Direct adaptive control offers better performance.
- The methodology has been extensively tested on an active suspension system.

Open problem:
Theoretical study of the plant – model mismatch.
Direct Adaptive Control (unknown $D_p$)

$$
\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d} B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \quad (*)
$$

We need to express $\varepsilon(t)$ as:

$$
\varepsilon(t+1) = \left[ Q(q^{-1}) - \hat{Q}(t+1, q^{-1}) \right] \Psi(t)
$$

Using: $MD_p + q^{-d} BQ = S_0$, $(*)$ becomes

$$
\varepsilon(t+1) = \left[ Q(q^{-1}) - \hat{Q}(t+1, q^{-1}) \right] \cdot \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + \frac{M(q^{-1}) D_p(q^{-1})}{P(q^{-1})} p(t+1)
$$

Instead of solving $MD_p + q^{-d} BQ = S_0$ search recursively for:

$$
\hat{Q}(t, q^{-1})^* = \arg \min_Q \sum_{i=0}^t \varepsilon^2[i, \hat{Q}]
$$

Details:

$$
\frac{M(q^{-1}) D_p(q^{-1})}{P(q^{-1})} p(t+1) = \frac{M(q^{-1}) N_p(q^{-1})}{P(q^{-1})} \delta(t+1)
$$

$$
\frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{q^{-d} B(q^{-1})}{P(q^{-1})} \cdot w(t+1)
$$