Output-Feedback Control of the Longitudinal Flight Dynamics Using Adaptative Backstepping

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Outline

1. Introduction
2. Problem statement
3. Controllers Design
   - Aerodynamic velocity controller
   - Flight-path-angle controller
4. Simulations
5. Conclusions & Future Work
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Introduction

Goal:
- Airplane flight control system for any operating point (Theory)
- Minimum aerodynamic knowledge (Theory)
- To fly an actual aircraft (Application)

Awkwardness:
- Nonlinear equations
- Aerodynamic models are difficult to obtain and often inaccurate
- Measured states $\Rightarrow$ outputs

Proposed approach:
- Consider only airplane longitudinal motion
- Controllers: the aerodynamic velocity + flight path angle
- Adaptive Backstepping scheme
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Aircraft model

Longitudinal aircraft equations of motion in wind axes

\[ \dot{V}_a = \frac{1}{m} \left( -D + F_T \cos \alpha - mg \sin \gamma \right) \]

\[ \dot{\gamma} = \frac{1}{mV_a} \left( L + F_T \sin \alpha - mg \cos \gamma \right) \]

\[ \dot{\theta} = q \]

\[ \dot{q} = \frac{M(\delta_e)}{I_y} \]

\[ \theta = \alpha + \gamma \]

With:

- \( m \) is the aircraft mass
- \( I_y \) is the aircraft inertia
- \( V_a \) is the aerodynamic velocity
- \( \alpha, \gamma \) and \( \theta \) are the attack, flight-path and pitch angles
- \( q \) is the pitch velocity
- \( D, L \) and \( M \) are the drag, lift and aerodynamic pitching moments
- \( F_T \) is the thrust

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\[
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\]

\[
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\]

\[\theta = \alpha + \gamma\]

Control problem

- Aerodynamic velocity $V_a$
- Flight path angle $\gamma$

Control inputs

- Thrust $F_T$.
- Elevator deflection $\delta_e$, through $M = f(\delta_e)$
Aerodynamic model

Aerodynamic coefficients definition:

\[
L = \frac{1}{2} \rho V_a^2 S C_L; \quad D = \frac{1}{2} \rho V_a^2 S C_D; \quad M = \frac{1}{2} \rho V_a^2 S \bar{c} C_m
\]

✓ We need a model for \( C_L \), \( C_D \) and \( C_m \) valid in all the flight envelope

Aerodynamic model used:

\[
C_L = f(\alpha)
\]
\[
C_D = C_{D_0} + k_1 \alpha + k_2 \alpha^2
\]
\[
C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} q + C_{m_{\delta_e}} \delta_e
\]

- The lift coefficient satisfies \( \alpha \cdot C_L(\alpha) \geq 0 \)
- Parabolic drag model satisfies \( C_D > 0 \)
- The aerodynamic moment coefficient satisfies \( C_{m_{\delta_e}} < 0 \)

Aerodynamic model of most conventional airplanes ("normal" flight conditions)
Control problem statement

Objectives

Improve the previous design of the flight-path-angle controller such that:
- NO knowledge of $C_L$, $C_D$ and $C_m$
- Drop assumption on trim angle of attack
- Change full-state feedback by output feedback
- (Global) regulation in any flight condition

Two independent controllers (for now):

**Aerodynamic velocity controller:**
- Thrust ($F_T$) as control signal
- Drag model coefficients unknown
- Adaptive control

**Measured variables:** $(V_a, \gamma, \theta, q)$

**Flight-path-angle controller:**
- Elevator ($\delta_e$) as control input
- ALL aerodynamic moment coefficients unknown
- Lift shape known
- Adaptive Backstepping approach
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Aerodynamic velocity controller

System:
\[
\dot{V}_a = \frac{1}{m} \left( -\frac{1}{2} \rho V_a^2 SC_D + F_T \cos \alpha - mg \sin \gamma \right)
\]

\[
\dot{z}_V = -\beta_1 \left( z_V^2 + V_{ref}^2 + 2z_V V_{ref} \right) \varphi(\alpha)^T \cdot \theta_V + \frac{F_T \cos \alpha}{m} - g \sin \gamma + \dot{V}_{ref}
\]

where

\[
z_V = V_a - V_{ref}, \quad \varphi(\alpha) = \begin{bmatrix} 1 & \alpha & \alpha^2 \end{bmatrix}^T, \quad \theta_V = \begin{bmatrix} C_{D0} & k_1 & k_2 \end{bmatrix}^T, \quad \beta_1 = \frac{\rho S}{2m}
\]

Adaptive-state feedback law:

\[
F_T = \frac{m}{\cos \alpha} \left( g \sin \gamma + \dot{V}_r + \beta_1 (z_V^2 + V_r^2) \varphi(\alpha)^T \cdot \hat{\theta}_V - \kappa V_1 z_V \right)
\]

\[
\dot{\hat{\theta}}_V = -\beta_1 z_V \left( z_V^2 + V_r^2 \right) \Gamma_V \varphi_V(\alpha)
\]

Lyapunov function:

\[
W_V = \frac{1}{2} z_V^2 + \frac{1}{2} \hat{\theta}_V^T \Gamma_V^{-1} \hat{\theta}_V
\]

Proposition: global boundedness of \(z_V\) and \(\hat{\theta}\), and convergence of \(z_V\) to zero.
Flight-path-angle controller I

System:
\[
\begin{align*}
\dot{\gamma} &= \frac{1}{mV_a} \left( L + F_T \sin \alpha - mg \cos \gamma \right) \\
\dot{\theta} &= q \\
\dot{q} &= M(\delta_e)/I_y
\end{align*}
\]

Assumption: \( \cos \gamma \approx \cos \gamma_{ref} \)

Property: \( (\alpha - \alpha_0)f(\alpha) \geq 0 \), 
\( \alpha_0 \): trim angle of attack, i.e. \( f(\alpha_0) = 0 \)

\( f(\alpha) \propto \frac{1}{2} \rho V_a^2 S C_L(\alpha) + F_T \sin \alpha - mg \cos \gamma_{ref} \)

\( f(\alpha) \) unknown \( \Rightarrow \) \( \alpha_0 \) is NOT computable

Assumptions for the control design:
- \( \dot{\gamma}_{ref} = 0 \) \( \Rightarrow \) regulation
- \( F_T \geq 0 \)

Error coordinates:
\[
\begin{align*}
z_1 &= \gamma - \gamma_{ref} \\
z_2 &= \theta - \gamma_{ref} - \alpha_0 \\
z_3 &= q
\end{align*}
\]

\[
\begin{align*}
\dot{\gamma} &= f(\alpha) = f(\theta - \gamma) \\
\dot{\theta} &= q \\
\dot{q} &= \frac{\rho V_a^2 S}{2I_y} \left( C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} q + C_{m_{\delta_e}} \delta_e \right)
\end{align*}
\]

\[
\eta(x) := f(x + \alpha_0), \; x \cdot \eta(x) \geq 0, \; x \in \mathbb{R}
\]

\[
\begin{align*}
\dot{z}_1 &= \eta(z_2 - z_1) \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= \beta_2 \left( C_{m_0} + C_{m_\alpha} (z_2 - z_1 + \alpha_0) + C_{m_q} z_3 + C_{m_{\delta_e}} \delta_e \right)
\end{align*}
\]
Control objective

To make the origin $z = 0$ globally asymptotically stable through the input $(\delta_e)$. Notice that $z = 0 \Leftrightarrow (\gamma, \theta, q) = (\gamma_{ref}, \theta_{ref}, 0)$

Adaptive Backstepping$^{++}$ scheme

C1. $\gamma_{ref}$ is a given reference.

C2. $\alpha_0$ is unknown and so, $\theta_{ref} := \alpha_0 + \gamma_{ref}$ too.

C3. $C_{m\delta_e}$ is unknown but assumed to be negative.

C4. The measurable output vector $y \in \mathbb{R}^3$ is defined as

$$y := \begin{bmatrix} \gamma - \gamma_{ref} \\ \alpha \\ q \end{bmatrix} \equiv \begin{bmatrix} z_1 \\ z_2 - z_1 + \alpha_0 \\ z_3 \end{bmatrix}$$

Remind: $\eta : \mathbb{R} \mapsto \mathbb{R}$ unknown, but with the property $x \cdot \eta(x) \geq 0$ (Krstic’95, Harkegard’03)
... after three steps yields

\[ \dot{z}_1 = \eta, \]
\[ \dot{z}_2 - k_{\gamma_1} z_1 = (z_3 + c_1 z_1) - c_1 z_1 + \kappa_{\gamma_1} \eta, \]
\[ \dot{z}_3 + c_1 z_1 = \beta_2 C_{m\delta_e} \left( \varphi_T \cdot \theta_\gamma + \delta_e \right) - \beta_2 \kappa_{\gamma_3} (z_3 + c_1 z_1) + c_1 \eta, \]

with \( \beta_2 = \frac{\rho V_a^2 S \bar{c}}{2 I_y}, \) \( \beta_{\delta_e} = \beta_2 C_{m\delta_e} \)

\[ \varphi_\gamma := \begin{bmatrix} 1 \\ y_2 \\ y_3 \\ \kappa_{\gamma_3} (y_3 + c_1 y_1) \end{bmatrix} = \begin{bmatrix} 1 \\ \alpha \\ q \\ \kappa_{\gamma_3} (q + c_1 (\gamma - \gamma_{ref})) \end{bmatrix}, \quad \theta_\gamma = \frac{1}{C_{m\delta_e}} \begin{bmatrix} C_{m_0} \\ C_{m_\alpha} \\ C_{m_q} \\ 1 \end{bmatrix} \]

Lyapunov function:

\[ W_3 = \frac{c_1}{2} z_1^2 + \int_0^{z_2-z_1} \eta(s) ds + \frac{c_3}{2} (z_3 + c_1 z_1)^2 + \frac{|C_{m_\delta_e}|}{2} \tilde{\theta}_\gamma^T \Gamma_\gamma^{-1} \tilde{\theta}_\gamma, \]

Prop.: The equilibrium manifold \( (\gamma, \theta, q, \hat{\theta}_\gamma) = (\gamma_{ref}, \theta_{ref}, 0, \hat{\theta}_\gamma^*) \) is GAS (\( \hat{\theta}_\gamma^* \) constant), with the adaptive output-feedback given by

\[ \delta_e = -\varphi_\gamma(y)^T \cdot \hat{\theta}_\gamma, \quad \dot{\hat{\theta}}_\gamma = -\frac{\beta_2}{c_1} (q + c_1 (\gamma - \gamma_{ref})) \Gamma_\gamma \varphi_\gamma(y). \]
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Simulations

Aerodynamic velocity & Céfiro

![Graph showing aerodynamic velocity over time](image)

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Simulations

**Flight-path angle & control inputs**

- Saturations in control signals included.

\[
F_T \in [4.9 \text{ N, } 117.6 \text{ N}] \\
\delta_e \in [-30^\circ, 30^\circ]
\]
Simulations

States & estimated parameters

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Conclusions & Future Work

General Conclusions

- A controller for a nonlinear longitudinal-aircraft dynamics is proposed
- Controller valid for any operating point
- NO knowledge of the aerodynamic model required ⇒ independent of the airplane
- Drop $\alpha_0$ by output feedback ⇒ independent of the lift curve
- Explicit and easy-to-implement control law

Technical comments & Future Work

- Output feedback with the cost of losing Exponential stability (IP)
- $\gamma_{ref}(t)$ ⇒ tracking. Proof?
- Related work: extremum-seeking?
- Include a propulsive model to use the throttle as control input
- Global stability of the equilibrium of the whole system
- Include constraints and saturations for a real flight test
Thanks!