

Voting power in the 25-EU under the Nice rules

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```
<<DiscreteMath`Combinatorica`
```

```
<<Graphics`Graphics`
```

```
<<Graphics`Graphics3D`
```

Banzhaf index

The function **banzhafG** computes the generating function given a list of integer weights. The function **banzhafIndexPlus** computes the total Banzhaf power index of player i by summing the appropriate coefficients in this generating function. Dividing each player's index by the sum of all the indices gives the Banzhaf power distribution.

```
banzhafG[weights_List]:=Times @@ (1+x^weights)
```

```
banzhafIndexPlus[i_,weights_List,q_]:=
Module[{delw,sw,g,coefi},
delw=Delete[weights,i];
sw=Apply[Plus,delw]+1;
g=banzhafG[delw];
coefi=CoefficientList[g,x];
Apply[Plus,coefi[[
Range[Max[1,q-weights[[i]]+1],Min[q,sw]]]]]
]
```

```
banzhafPowerPlus[weights_List,q_]:=
#/(Plus @@ #)& @ Table[banzhafIndexPlus[i,weights,q],
{i,Length[weights]}]
```

```
critical[weights_List,q_]:=
Table[banzhafIndexPlus[i,weights,q],{i,Length[weights]}]
```

Shapley-Shubik index

The number of coalitions of weight k and size j is the coefficient of $x^k z^j$ in the generating function $g(x,y)$ for the Shapley-Shubik index. The function **ssG** gives the polynomial $g(x,y)$. The function **ssPowerPlus** computes the Shapley Shubik power distribution. We suppose that the simple game is superadditive, that is, winning disjoint coalitions are not possible.

```
ssG[weights_List]:=Times @@ (1+z x^weights)
```

```
ssPowerPlus[weights_List,q_Integer]:=
Module[{n=Length[weights],delw,sw,g,coefi,gg},
Table[delw=Delete[weights,i];
sw=Apply[Plus,delw]+1;
g=ssG[delw];
coefi=CoefficientList[g,x];
gg=Apply[Plus,coefi[[
Range[Max[1,q-weights[[i]]+1,Min[q,sw]]]]]];
Sum[Coefficient[gg,z^j] j! (n-j-1)!,{j,n-1}],
{i,n}]/n!]
```

Weighting of votes in the 27 EU with the Nice rule

```
countriesEU25={"Germany","United Kingdom","France","Italy","Spain","Poland",
"Netherlands","Greece","Czech Republic","Belgium","Hungary","Portugal","Sweedeen",
"Austria","Slovak Republic","Denmark","Finland","Ireland","Lithuania","Latvia",
"Slovenia","Estonia","Cyprus","Luxembourg","Malta"};
```

```
popEU25={82.038,59.247,58.966,57.612,40.394,38.667,15.760,10.533,10.290,10.213,
10.092,9.980,8.854,8.082,5.393,5.313,5.160,3.744,3.701,2.439,1.978,1.446,0.752,
0.429,0.379};
```

```
indexpopEU25=SetPrecision[%/Plus @@ %],3]
```

```
{0.182, 0.131, 0.131, 0.128, 0.0895, 0.0856, 0.0349,
0.0233, 0.0228, 0.0226, 0.0224, 0.0221, 0.0196,
0.0179, 0.0119, 0.0118, 0.0114, 0.00829, 0.00820,
0.00540, 0.00438, 0.00320, 0.00167, 0.000950,
0.000839}
```

```
proporEU25 = Round[(popEU25/Plus @@ popEU25) 321.2]
```

```
{58, 42, 42, 41, 29, 28, 11, 7, 7, 7, 7, 7, 6, 6, 4, 4,
4, 3, 3, 2, 1, 1, 1, 0, 0}
```

```
Plus @@ %
```

```
321
```

```
propor25data=Transpose[{popEU25,proporEU25}]
```

```
{{82.038, 58}, {59.247, 42}, {58.966, 42},
{57.612, 41}, {40.394, 29}, {38.667, 28},
{15.76, 11}, {10.533, 7}, {10.29, 7}, {10.213, 7},
{10.092, 7}, {9.98, 7}, {8.854, 6}, {8.082, 6},
{5.393, 4}, {5.313, 4}, {5.16, 4}, {3.744, 3},
{3.701, 3}, {2.439, 2}, {1.978, 1}, {1.446, 1},
{0.752, 1}, {0.429, 0}, {0.379, 0}}
```

```

votesEU25={29,29,29,29,27,27,13,12,12,12,12,12,
10,10,7,7,7,7,7,4,4,4,4,4,3};
votes25Index=SetPrecision[(%/Plus @@ %),3]

{0.0903, 0.0903, 0.0903, 0.0903, 0.0841, 0.0841,
0.0405, 0.0374, 0.0374, 0.0374, 0.0374, 0.0374,
0.0312, 0.0312, 0.0218, 0.0218, 0.0218, 0.0218,
0.0218, 0.0125, 0.0125, 0.0125, 0.0125, 0.0125,
0.00935}

Plus @@ votesEU25

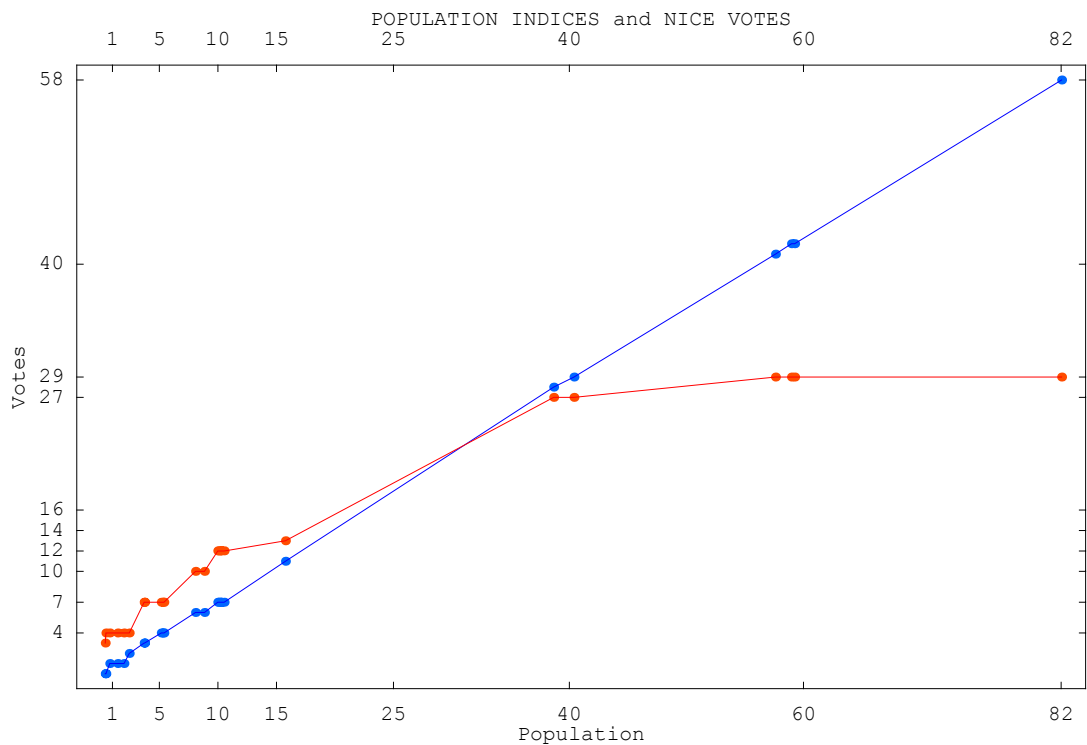
321

EU25nicedata=Transpose[{popEU25,votesEU25}]

{{82.038, 29}, {59.247, 29}, {58.966, 29},
{57.612, 29}, {40.394, 27}, {38.667, 27},
{15.76, 13}, {10.533, 12}, {10.29, 12}, {10.213, 12},
{10.092, 12}, {9.98, 12}, {8.854, 10}, {8.082, 10},
{5.393, 7}, {5.313, 7}, {5.16, 7}, {3.744, 7},
{3.701, 7}, {2.439, 4}, {1.978, 4}, {1.446, 4},
{0.752, 4}, {0.429, 4}, {0.379, 3}}

DisplayTogether[ListPlot[EU25nicedata,Frame->True,
FrameLabel->{"Population","Votes"},
"POPULATION INDICES and NICE VOTES","",
FrameTicks->{{1,5,10,15,25,40,60,82},
{4,7,10,12,14,16,27,29,40,58}},
PlotStyle->{Hue[0.05],PointSize[0.009]}],
ListPlot[propor25data,PlotStyle->{Hue[0.6],PointSize[0.009]}],
ListPlot[propor25data,PlotJoined->True,PlotStyle->RGBColor[0,0,1]],
ListPlot[EU25nicedata,PlotJoined->True,PlotStyle->RGBColor[1,0,0]]];

```



Banzhaf index for the meet of two games

```
banzhafTwoG[weights_List,pop_List]:=
Times @@ (1+x^weights y^pop)
```

```
banzhafTwoG[{4,6,8},{2,3,4}]
```

$$(1 + x^4 y^2) (1 + x^6 y^3) (1 + x^8 y^4)$$

```
banzhafTwoIndex[i_,weights_List,pop_List,q_,p_]:=
Module[{g,coefi,n,m,s1,s2},
g=banzhafTwoG[Delete[weights,i],Delete[pop,i]];
n=Exponent[g,x]+1; m=Exponent[g,y]+1;
coefi=CoefficientList[g,{x,y}]/.{}->Table[0,{m}];
s1=Apply[Plus,Flatten[coefi[[
Range[Max[1,q-weights[[i]]+1],n],
Range[Max[1,p-pop[[i]]+1],m]]]]];
s2=If[((q+1)>n) || ((p+1)>m),0,
Apply[Plus,Flatten[coefi
[[Range[q+1,n],Range[p+1,m]]]]]]];
s1-s2]
```

```
criticalTwo[weights_List,pop_List,q_,p_]:=
Table[banzhafTwoIndex[i,weights,pop,q,p],{i,Length[weights]}]
```

```
banzhafTwoPower[weights_List,pop_List,q_,p_]:=
#/(Plus @@ #)& @ Table[
banzhafTwoIndex[i,weights,pop,q,p],
{i,Length[weights]}]
```

Shapley-Shubik index for the meet of two games

```
ssTwoG[weights_List,pop_List]:=Times @@ (1+x^weights y^pop z)
```

```
ssTwoPower[weights_List,pop_List,q_,p_]:=
Module[{n=Length[weights],g,dw,dp,s1,s2,gg,coefi},
Table[g=ssTwoG[Delete[weights,i],Delete[pop,i]];
dw=Exponent[g,x]+1; dp=Exponent[g,y]+1;
coefi=CoefficientList[g,{x,y}]/.{}->Table[0,{dp}];
s1=Apply[Plus,Flatten[coefi[[
Range[Max[1,q-weights[[i]]+1],dw],
Range[Max[1,p-pop[[i]]+1],dp]]]]];
s2=If[((q+1)>dw) || ((p+1)>dp),0,
Apply[Plus,Flatten[coefi[[
Range[q+1,dw],Range[p+1,dp]]]]]]];
gg=s1-s2;
Sum[Coefficient[gg,z,j] j! (n-j-1)!,{j,n-1}]/n!,
{i,n}]]
```

Banzhaf and Shapley indices for the meet of three games

```

banzhafThreeG[weights_List,pop_List,members_List] :=
Times @@ (1 + x^weights*y^pop*z^members)

banzhaf3Index[i_,weights_List,pop_List,members_List,q_,p_,m_] :=
Module[{n = Length[weights],g,dw,dp,dm,s1,s2,gg,coefi,delwe,delpo,delm},
delwe = Delete[weights,i]; delpo = Delete[pop,i]; delm = Delete[members,i];
g = banzhafThreeG[delwe, delpo, delm];
dw = Plus @@ delwe + 1; dp = Plus @@ delpo + 1; dm = Plus @@ delm + 1;
coefi = CoefficientList[g, {x, y, z}] /. {} -> Table[0, {dp}, {dw}];
s1 = Plus @@ Flatten[coefi[[Range[Max[1, q - weights[[i]] + 1], dw],
Range[Max[1, p - pop[[i]] + 1], dp],
Range[Max[1, m - members[[i]] + 1], dm]]];
s2 = If[q + 1 > dw || p + 1 > dp || m + 1 > dm, 0,
Plus @@ Flatten[coefi[[Range[q + 1, dw],Range[p + 1, dp],
Range[m + 1, dm]]]]]; gg = s1 - s2]

banzhaf3swings[weights_List,pop_List,members_List,q_,p_,m_] :=
Table[banzhaf3Index[i,weights,pop,members,q,p,m],
{i, Length[weights]}]

banzhaf3Power[weights_List,pop_List,members_List,q_,p_,m_] :=
(#1/Plus @@ #1 & ) [Table[banzhaf3Index[i,weights,pop,members,q,p,m],
{i, Length[weights]}]]

Shapley3G[weights_List,pop_List,members_List] :=
Times @@ (1 + x^weights*y^pop*z^members*t)

Shapley3Index[weights_List,pop_List,members_List,q_,p_,m_] :=
Module[{n=Length[weights],g,dw,dp,dm,s1,s2,gg,coefi,delwe,delpo,delm},
Table[delwe=Delete[weights,i];delpo=Delete[pop,i];delm=Delete[members,i];
g=Shapley3G[delwe,delpo,delm];
dw = Plus @@ delwe+1; dp = Plus @@ delpo+1; dm = Plus @@ delm+1;
coefi = CoefficientList[g, {x, y, z}] /. {} -> Table[0, {dp}, {dw}];
s1 = Plus @@ Flatten[coefi[[Range[Max[1, q-weights[[i]]+1], dw],
Range[Max[1,p-pop[[i]]+1],dp],Range[Max[1,m-members[[i]]+1],dm]]];
s2 = If[q + 1 > dw || p + 1 > dp || m + 1 > dm, 0,
Plus @@ Flatten[coefi[[Range[q+1,dw],Range[p+1,dp],Range[m+1,dm]]]]];
gg = s1 - s2; Sum[Coefficient[gg,t,j] j! (n-j-1)!,{j,0,n-1}]/n!,{i,n}]]

```

Nice swings in the 25 EU

The total number of coalitions in the 27 European Union is 33 554 432. We calculate the number of winning coalitions to which the country i is critical, that is, his defection implies that the coalition to become losing.

```
members25=Table[1,{25}];
```

```
swings1=critical[votesEU25,232]
```

```
{924234, 924234, 924234, 924234, 876978, 876978,
 456480, 422150, 422150, 422150, 422150, 422150,
 353022, 353022, 248870, 248870, 248870, 248870,
 248870, 142854, 142854, 142854, 142854, 142854,
 106590}
```

```
ban1EU25=SetPrecision[% / Plus @@ % ,3]
```

```
{0.0857, 0.0857, 0.0857, 0.0857, 0.0813, 0.0813,
 0.0423, 0.0391, 0.0391, 0.0391, 0.0391, 0.0391,
 0.0327, 0.0327, 0.0231, 0.0231, 0.0231, 0.0231,
 0.0231, 0.0132, 0.0132, 0.0132, 0.0132, 0.0132,
 0.00988}
```

```
Plus @@ swings1
```

```
10789376
```

```
swingsTwo1=criticalTwo[votesEU25,members25,232,13]
```

```
{923570, 923570, 923570, 923570, 876314, 876314,
 456252, 421938, 421938, 421938, 421938, 421938,
 353124, 353124, 249302, 249302, 249302, 249302,
 249302, 143446, 143446, 143446, 143446, 143446,
 107202}
```

```
ban2aEU25=SetPrecision[% / Plus @@ % ,3]
```

```
{0.0856, 0.0856, 0.0856, 0.0856, 0.0812, 0.0812,
 0.0423, 0.0391, 0.0391, 0.0391, 0.0391, 0.0391,
 0.0327, 0.0327, 0.0231, 0.0231, 0.0231, 0.0231,
 0.0231, 0.0133, 0.0133, 0.0133, 0.0133, 0.0133,
 0.00994}
```

```
Plus @@ swingsTwo1
```

```
10790040
```

```
swingsTwo2=criticalTwo[votesEU25,members25,232,17]
```

```
{532089, 532089, 532089, 532089, 508399, 508399,
 319185, 309505, 309505, 309505, 309505, 309505,
 284963, 284963, 249653, 249653, 249653, 249653,
 249653, 214637, 214637, 214637, 214637, 214637,
 202577}
```

```
ban2bEU25=SetPrecision[% / Plus @@ % ,3]
```

```
{0.0656, 0.0656, 0.0656, 0.0656, 0.0627, 0.0627,  
 0.0394, 0.0382, 0.0382, 0.0382, 0.0382, 0.0382,  
 0.0352, 0.0352, 0.0308, 0.0308, 0.0308, 0.0308,  
 0.0308, 0.0265, 0.0265, 0.0265, 0.0265, 0.0265,  
 0.0250}
```

```
Plus @@ swingsTwo2
```

```
8105817
```

```
weigthEU25=Round[(popEU25/Plus @@ popEU25) 1000]
```

```
{182, 131, 131, 128, 89, 86, 35, 23, 23, 23, 22, 22,  
 20, 18, 12, 12, 11, 8, 8, 5, 4, 3, 2, 1, 1}
```

```
Plus @@ %
```

```
1000
```

```
swings3a=banzhaf3swings[votesEU25,weigthEU25,members25,232,620,13]
```

```
{923614, 923558, 923558, 923558, 876310, 876310,  
 456206, 421892, 421892, 421892, 421892, 421892,  
 353078, 353078, 249256, 249256, 249256, 249256,  
 249256, 143412, 143412, 143412, 143412, 143412,  
 107168}
```

```
ban3aEU25=SetPrecision[swings3a / Plus @@ swings3a ,3]
```

```
{0.0856, 0.0856, 0.0856, 0.0856, 0.0812, 0.0812,  
 0.0423, 0.0391, 0.0391, 0.0391, 0.0391, 0.0391,  
 0.0327, 0.0327, 0.0231, 0.0231, 0.0231, 0.0231,  
 0.0231, 0.0133, 0.0133, 0.0133, 0.0133, 0.0133,  
 0.00993}
```

```
Plus @@ swings3a
```

```
10789238
```

```
swings3b=banzhaf3swings[votesEU25,weigthEU25,members25,232,620,17]
```

```
{532133, 532077, 532077, 532077, 508395, 508395,  
 319139, 309459, 309459, 309459, 309459, 309459,  
 284917, 284917, 249607, 249607, 249607, 249607,  
 249607, 214603, 214603, 214603, 214603, 214603,  
 202543}
```

```
ban3bEU25=SetPrecision[swings3b / Plus @@ swings3b ,3]
```

```
{0.0657, 0.0656, 0.0656, 0.0656, 0.0627, 0.0627,  
 0.0394, 0.0382, 0.0382, 0.0382, 0.0382, 0.0382,  
 0.0352, 0.0352, 0.0308, 0.0308, 0.0308, 0.0308,  
 0.0308, 0.0265, 0.0265, 0.0265, 0.0265, 0.0265,  
 0.0250}
```

Plus @@ swings3b

8105015

```
TableForm[Transpose[{swings1,swingsTwo1,swings3a,swings3a-swings1}],
TableHeadings->{countriesEU25,
{"Swings V","Swings V+M","Swings V+M+P","Difference"}}]
```

	Swings V	Swings V+M	Swings V+M+P	Difference
Germany	924234	923570	923614	-620
United Kingdom	924234	923570	923558	-676
France	924234	923570	923558	-676
Italy	924234	923570	923558	-676
Spain	876978	876314	876310	-668
Poland	876978	876314	876310	-668
Netherlands	456480	456252	456206	-274
Greece	422150	421938	421892	-258
Czech Republic	422150	421938	421892	-258
Belgium	422150	421938	421892	-258
Hungary	422150	421938	421892	-258
Portugal	422150	421938	421892	-258
Sweedeen	353022	353124	353078	56
Austria	353022	353124	353078	56
Slovak Republic	248870	249302	249256	386
Denmark	248870	249302	249256	386
Finland	248870	249302	249256	386
Ireland	248870	249302	249256	386
Lithuania	248870	249302	249256	386
Latvia	142854	143446	143412	558
Slovenia	142854	143446	143412	558
Estonia	142854	143446	143412	558
Cyprus	142854	143446	143412	558
Luxembourg	142854	143446	143412	558
Malta	106590	107202	107168	578

```
TableForm[Transpose[{swings1, swingsTwo2, swings3b, swings3b-swings1}],
TableHeadings->{countriesEU25,
{"Sw V", "Sw V+(2/3)", "Sw V+(2/3)+P", "Difference"}]]
```

	Sw V	Sw V+(2/3)	Sw V+(2/3)+P	Difference
Germany	924234	532089	532133	-392101
United Kingdom	924234	532089	532077	-392157
France	924234	532089	532077	-392157
Italy	924234	532089	532077	-392157
Spain	876978	508399	508395	-368583
Poland	876978	508399	508395	-368583
Netherlands	456480	319185	319139	-137341
Greece	422150	309505	309459	-112691
Czech Republic	422150	309505	309459	-112691
Belgium	422150	309505	309459	-112691
Hungary	422150	309505	309459	-112691
Portugal	422150	309505	309459	-112691
Sweedeen	353022	284963	284917	-68105
Austria	353022	284963	284917	-68105
Slovak Republic	248870	249653	249607	737
Denmark	248870	249653	249607	737
Finland	248870	249653	249607	737
Ireland	248870	249653	249607	737
Lithuania	248870	249653	249607	737
Latvia	142854	214637	214603	71749
Slovenia	142854	214637	214603	71749
Estonia	142854	214637	214603	71749
Cyprus	142854	214637	214603	71749
Luxembourg	142854	214637	214603	71749
Malta	106590	202577	202543	95953

Voting power with the Nice rules

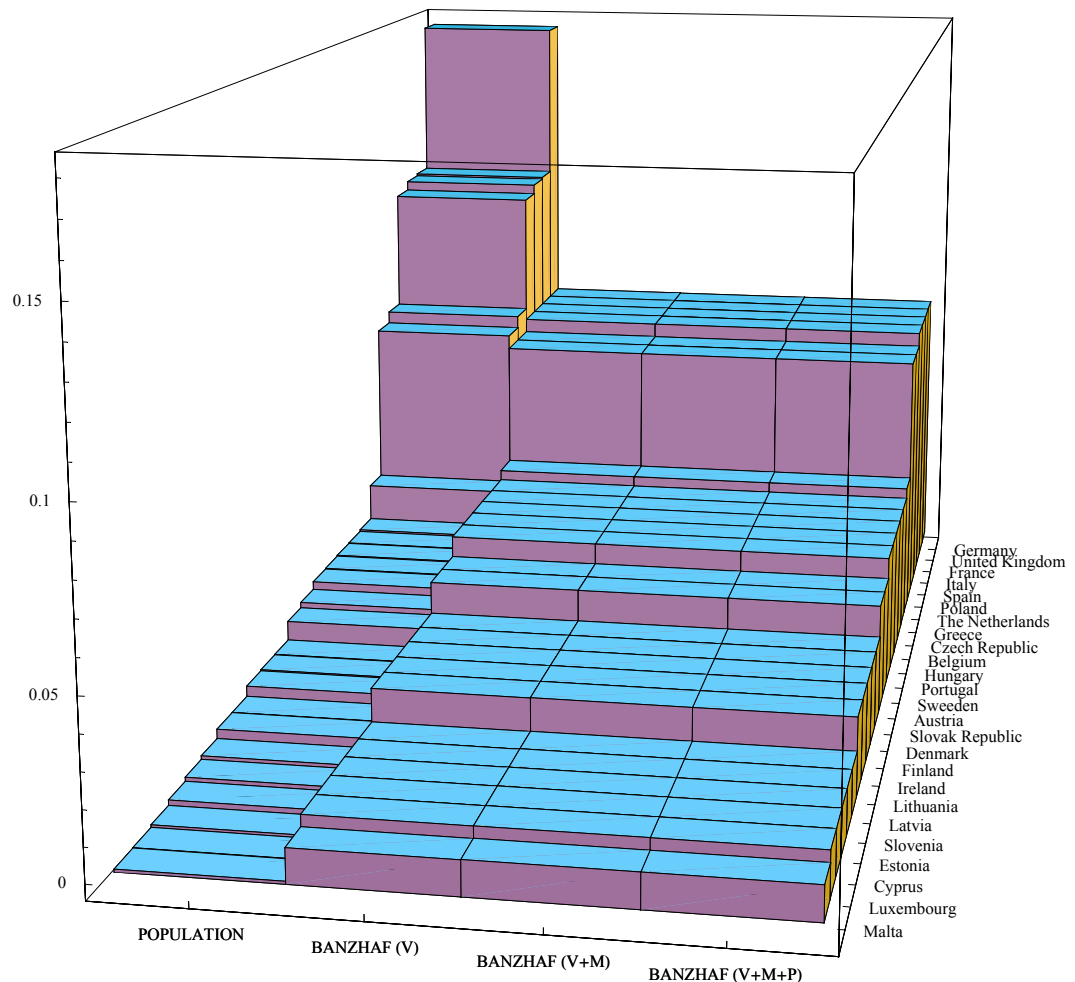
```
TableForm[Transpose[{{indexpopEU25,ban1EU25,ban2aEU25,ban3aEU25}},
TableHeadings->{countriesEU25,{"People","Banzhaf V",
"Banzhaf V+M","Banzhaf V+M+P"}}]]
```

	People	Banzhaf V	Banzhaf V+M	Banzhaf V+M+P
Germany	0.182	0.0857	0.0856	0.0856
United Kingdom	0.131	0.0857	0.0856	0.0856
France	0.131	0.0857	0.0856	0.0856
Italy	0.128	0.0857	0.0856	0.0856
Spain	0.0895	0.0813	0.0812	0.0812
Poland	0.0856	0.0813	0.0812	0.0812
Netherlands	0.0349	0.0423	0.0423	0.0423
Greece	0.0233	0.0391	0.0391	0.0391
Czech Republic	0.0228	0.0391	0.0391	0.0391
Belgium	0.0226	0.0391	0.0391	0.0391
Hungary	0.0224	0.0391	0.0391	0.0391
Portugal	0.0221	0.0391	0.0391	0.0391
Sweeden	0.0196	0.0327	0.0327	0.0327
Austria	0.0179	0.0327	0.0327	0.0327
Slovak Republic	0.0119	0.0231	0.0231	0.0231
Denmark	0.0118	0.0231	0.0231	0.0231
Finland	0.0114	0.0231	0.0231	0.0231
Ireland	0.00829	0.0231	0.0231	0.0231
Lithuania	0.00820	0.0231	0.0231	0.0231
Latvia	0.00540	0.0132	0.0133	0.0133
Slovenia	0.00438	0.0132	0.0133	0.0133
Estonia	0.00320	0.0132	0.0133	0.0133
Cyprus	0.00167	0.0132	0.0133	0.0133
Luxembourg	0.000950	0.0132	0.0133	0.0133
Malta	0.000839	0.00988	0.00994	0.00993

```

banzhafEU25A=BarChart3D[
{Reverse[indexpopEU25],Reverse[ban1EU25],
Reverse[ban2aEU25],Reverse[ban3aEU25]},
Boxed -> True, BoxRatios -> {0.5,1,0.5},
ViewPoint->{0.4,-2.2,0.5},Ticks ->{{1,"POPULATION"},
{2,"BANZHAF (V)"},{3,"BANZHAF (V+M)"},{4,"BANZHAF (V+M+P)"},
{{1,"Malta"},{2,"Luxembourg"},{3,"Cyprus"},
{4,"Estonia"},{5,"Slovenia"},{6,"Latvia"},
{7,"Lithuania"},{8,"Ireland"},{9,"Finland"},
{10,"Denmark"},{11,"Slovak Republic"},
{12,"Austria"},{13,"Sweedeen"},
{14,"Portugal"},{15,"Hungary"},{16,"Belgium"},
{17,"Czech Republic"},{18,"Greece"},
{19,"The Netherlands"},{20,"Poland"},{21,"Spain"},
{22,"Italy"},{23,"France"},{24,"United Kingdom"},
{25,"Germany"}},Automatic),
DefaultFont->{"Times",8}];

```



Voting power with the two thirds Nice rules

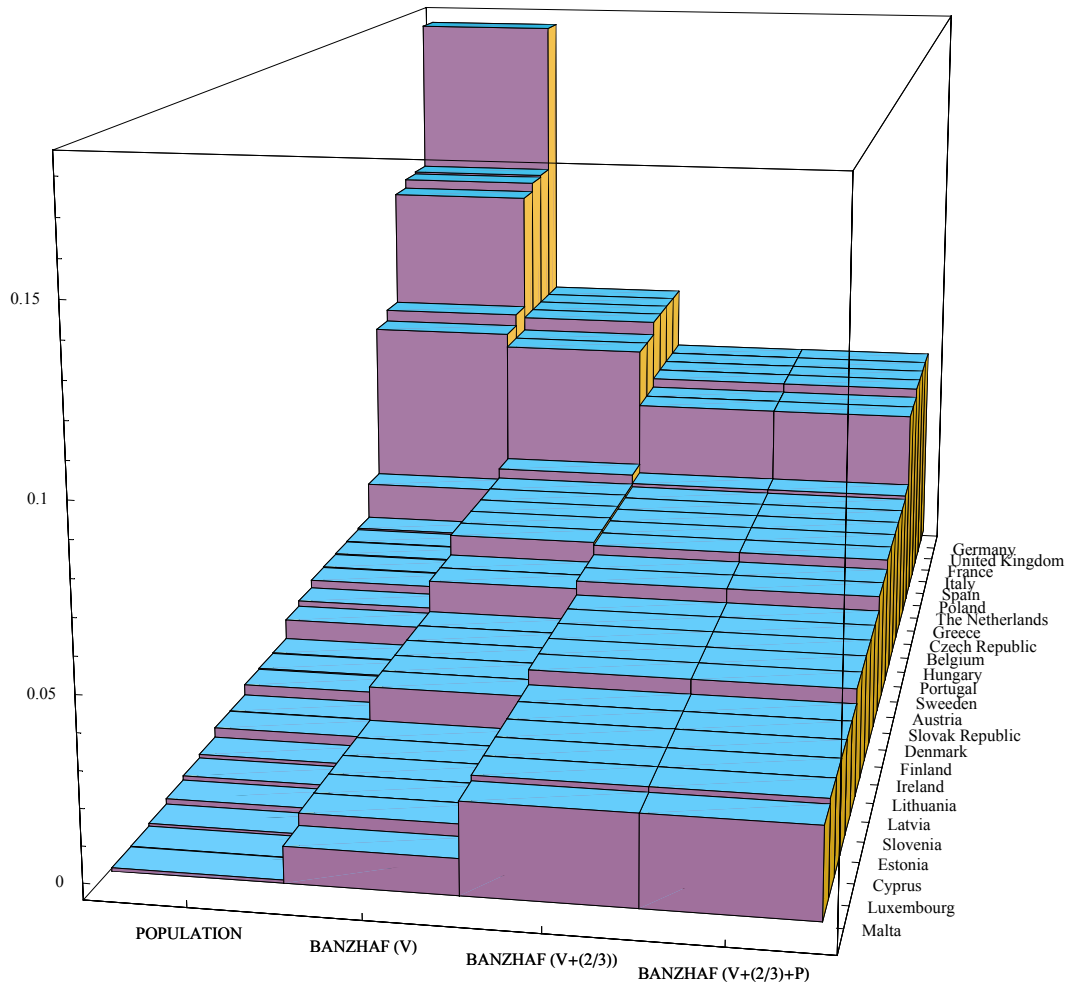
**TableForm[Transpose[{{indexpopEU25,ban1EU25,ban2bEU25,ban3bEU25}},
TableHeadings->{countriesEU25,{"People","Ban V",
"Ban V+(2/3)M","Ban V+(2/3)M+P"}}]]**

	People	Ban V	Ban V+(2/3)M	Ban V+(2/3)M+P
Germany	0.182	0.0857	0.0656	0.0657
United Kingdom	0.131	0.0857	0.0656	0.0656
France	0.131	0.0857	0.0656	0.0656
Italy	0.128	0.0857	0.0656	0.0656
Spain	0.0895	0.0813	0.0627	0.0627
Poland	0.0856	0.0813	0.0627	0.0627
Netherlands	0.0349	0.0423	0.0394	0.0394
Greece	0.0233	0.0391	0.0382	0.0382
Czech Republic	0.0228	0.0391	0.0382	0.0382
Belgium	0.0226	0.0391	0.0382	0.0382
Hungary	0.0224	0.0391	0.0382	0.0382
Portugal	0.0221	0.0391	0.0382	0.0382
Sweedden	0.0196	0.0327	0.0352	0.0352
Austria	0.0179	0.0327	0.0352	0.0352
Slovak Republic	0.0119	0.0231	0.0308	0.0308
Denmark	0.0118	0.0231	0.0308	0.0308
Finland	0.0114	0.0231	0.0308	0.0308
Ireland	0.00829	0.0231	0.0308	0.0308
Lithuania	0.00820	0.0231	0.0308	0.0308
Latvia	0.00540	0.0132	0.0265	0.0265
Slovenia	0.00438	0.0132	0.0265	0.0265
Estonia	0.00320	0.0132	0.0265	0.0265
Cyprus	0.00167	0.0132	0.0265	0.0265
Luxembourg	0.000950	0.0132	0.0265	0.0265
Malta	0.000839	0.00988	0.0250	0.0250

```

banzhafEU25B=BarChart3D[
  {Reverse[indexpopEU25],Reverse[ban1EU25],
  Reverse[ban2bEU25],Reverse[ban3bEU25]},
  Boxed -> True, BoxRatios -> {0.5,1,0.5},
  ViewPoint->{0.4,-2.2,0.5},Ticks ->{{1,"POPULATION"},
  {2,"BANZHAF (V)},{3,"BANZHAF (V+(2/3))",{4,"BANZHAF (V+(2/3)+P)}},
  {{1,"Malta"},{2,"Luxembourg"},{3,"Cyprus"},
  {4,"Estonia"},{5,"Slovenia"},{6,"Latvia"},
  {7,"Lithuania"},{8,"Ireland"},{9,"Finland"},
  {10,"Denmark"},{11,"Slovak Republic"},
  {12,"Austria"},{13,"Sweedden"},
  {14,"Portugal"},{15,"Hungary"},{16,"Belgium"},
  {17,"Czech Republic"},{18,"Greece"},
  {19,"The Netherlands"},{20,"Poland"},{21,"Spain"},
  {22,"Italy"},{23,"France"},{24,"United Kingdom"},
  {25,"Germany"}},Automatic),
  DefaultFont->{"Times",8}];

```



```

weightEU=Round[(popEU25/Plus @@ popEU25) 102]

```

```

{19, 13, 13, 13, 9, 9, 4, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0}

```

Plus @@ %

100

sha1EU25 = N[ssPowerPlus[votesEU25, 232]]

{0.0929262, 0.0929262, 0.0929262, 0.0929262, 0.0861356, 0.0861356, 0.0398291,
0.0364794, 0.0364794, 0.0364794, 0.0364794, 0.0364794, 0.0302405, 0.0302405,
0.0209839, 0.0209839, 0.0209839, 0.0209839, 0.0209839, 0.011892, 0.011892,
0.011892, 0.011892, 0.011892, 0.00893768}

sha2aEU25 = N[ssTwoPower[votesEU25, members25, 232, 13]]

{0.0929155, 0.0929155, 0.0929155, 0.0929155, 0.086125, 0.086125, 0.0398252,
0.0364757, 0.0364757, 0.0364757, 0.0364757, 0.0364757, 0.0302417, 0.0302417,
0.0209901, 0.0209901, 0.0209901, 0.0209901, 0.0209901, 0.0119007, 0.0119007,
0.0119007, 0.0119007, 0.0119007, 0.00894671}

sha3aEU25 = N[Shapley3Index[votesEU25, weightEU, members25, 232, 62, 13]]

{0.09516, 0.0937271, 0.0937271, 0.0937271, 0.0866436, 0.0866436, 0.0397405,
0.0360908, 0.0360908, 0.0360908, 0.0360908, 0.0360908, 0.029946, 0.029946,
0.0206343, 0.0206343, 0.0206343, 0.0206343, 0.0206343, 0.011816, 0.0116503,
0.0116503, 0.0116503, 0.0116503, 0.00869633}

```
SetPrecision[TableForm[Transpose[{indexpopEU25, sha1EU25, sha2aEU25,
  sha3aEU25}], TableHeadings -> {countriesEU25, {"People", "Shapley V",
  "Shapley V+M", "Shapley V+M+P"}}], 3]
```

	People	Shapley V	Shapley V+M	Shapley V+M+P
Germany	0.182	0.0929	0.0929	0.0952
United Kingdom	0.131	0.0929	0.0929	0.0937
France	0.131	0.0929	0.0929	0.0937
Italy	0.128	0.0929	0.0929	0.0937
Spain	0.0895	0.0861	0.0861	0.0866
Poland	0.0856	0.0861	0.0861	0.0866
Netherlands	0.0349	0.0398	0.0398	0.0397
Greece	0.0233	0.0365	0.0365	0.0361
Czech Republic	0.0228	0.0365	0.0365	0.0361
Belgium	0.0226	0.0365	0.0365	0.0361
Hungary	0.0224	0.0365	0.0365	0.0361
Portugal	0.0221	0.0365	0.0365	0.0361
Sweedeen	0.0196	0.0302	0.0302	0.0299
Austria	0.0179	0.0302	0.0302	0.0299
Slovak Republic	0.0119	0.0210	0.0210	0.0206
Denmark	0.0118	0.0210	0.0210	0.0206
Finland	0.0114	0.0210	0.0210	0.0206
Ireland	0.00829	0.0210	0.0210	0.0206
Lithuania	0.00820	0.0210	0.0210	0.0206
Latvia	0.00540	0.0119	0.0119	0.0118
Slovenia	0.00438	0.0119	0.0119	0.0117
Estonia	0.00320	0.0119	0.0119	0.0117
Cyprus	0.00167	0.0119	0.0119	0.0117
Luxembourg	0.000950	0.0119	0.0119	0.0117
Malta	0.000839	0.00894	0.00895	0.00870

```
sha2bEU25 = N[ssTwoPower[votesEU25, members25, 232, 17]]
```

```
{0.0837547, 0.0837547, 0.0837547, 0.0837547, 0.0777974, 0.0777974, 0.0387623,
  0.0362392, 0.0362392, 0.0362392, 0.0362392, 0.0362392, 0.0314733, 0.0314733,
  0.0244541, 0.0244541, 0.0244541, 0.0244541, 0.0244541, 0.0177208, 0.0177208,
  0.0177208, 0.0177208, 0.0156068}
```

```
sha3bEU25 = N[Shapley3Index[votesEU25, weigthEU, members25, 232, 62, 17]]
```

```
{0.0859992, 0.0845663, 0.0845663, 0.0845663, 0.078316, 0.078316, 0.0386776,
  0.0358544, 0.0358544, 0.0358544, 0.0358544, 0.0358544, 0.0311776, 0.0311776,
  0.0240982, 0.0240982, 0.0240982, 0.0240982, 0.0240982, 0.0176361, 0.0174704,
  0.0174704, 0.0174704, 0.0153564}
```

```
SetPrecision[TableForm[Transpose[{indexpopEU25, sha1EU25, sha2bEU25,
  sha3bEU25}], TableHeadings -> {countriesEU25, {"People", "Shapley V",
  "Shapley V+(2M/3)", "Shapley V+(2M/3)+P"}}], 3]
```

	People	Shapley V	Shapley V+(2M/3)	Shapley V+(2M/3)+P
Germany	0.182	0.0929	0.0838	0.0860
United Kingdom	0.131	0.0929	0.0838	0.0846
France	0.131	0.0929	0.0838	0.0846
Italy	0.128	0.0929	0.0838	0.0846
Spain	0.0895	0.0861	0.0778	0.0783
Poland	0.0856	0.0861	0.0778	0.0783
Netherlands	0.0349	0.0398	0.0388	0.0387
Greece	0.0233	0.0365	0.0362	0.0359
Czech Republic	0.0228	0.0365	0.0362	0.0359
Belgium	0.0226	0.0365	0.0362	0.0359
Hungary	0.0224	0.0365	0.0362	0.0359
Portugal	0.0221	0.0365	0.0362	0.0359
Sweedden	0.0196	0.0302	0.0315	0.0312
Austria	0.0179	0.0302	0.0315	0.0312
Slovak Republic	0.0119	0.0210	0.0245	0.0241
Denmark	0.0118	0.0210	0.0245	0.0241
Finland	0.0114	0.0210	0.0245	0.0241
Ireland	0.00829	0.0210	0.0245	0.0241
Lithuania	0.00820	0.0210	0.0245	0.0241
Latvia	0.00540	0.0119	0.0177	0.0176
Slovenia	0.00438	0.0119	0.0177	0.0175
Estonia	0.00320	0.0119	0.0177	0.0175
Cyprus	0.00167	0.0119	0.0177	0.0175
Luxembourg	0.000950	0.0119	0.0177	0.0175
Malta	0.000839	0.00894	0.0156	0.0154