

Shapley-Shubik index in the EU

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In[1]:= <<DiscreteMath`Combinatorica`

In[2]:= <<Cooperat`Cooperat`

In[3]:= <<Graphics`Graphics`

In[4]:= <<Graphics`Graphics3D`

In[5]:= population={80.61,57.96,57.53,56.93,39.11,15.24,10.35,
10.07,9.86,8.69,7.91,5.18,5.06,3.56,0.4};
PUE=SetPrecision[%/Plus @@ %,3]

Out[6]= {0.219, 0.157, 0.156, 0.155, 0.106, 0.0414, 0.0281, 0.0273,
0.0268, 0.0236, 0.0215, 0.0141, 0.0137, 0.00966, 0.00109}

In[7]:= votosUE={10,10,10,10,8,5,5,5,5,4,4,3,3,3,2};
VUE=SetPrecision[%/Plus @@ %,3]

Out[8]= {0.115, 0.115, 0.115, 0.115, 0.0920, 0.0575, 0.0575,
0.0575, 0.0575, 0.0460, 0.0460, 0.0345, 0.0345, 0.0345,
0.0230}

In[9]:= ssG[weights_List]:=Times @@ (1+z x^weights)

In[10]:= Length[ssG[votosUE]//Expand]

Out[10]= 338

In[11]:= ssPowerPlus[weights_List,q_Integer]:=
Module[{n=Length[weights],delw,sw,g,gg,coefi},
Table[delw=Delete[weights,i];
g=ssG[delw];
sw=Apply[Plus,delw]+1;
coefi=CoefficientList[g,x];
gg=Apply[Plus,coefi[[
Range[Max[1,q-weights[[i]]+1,Min[q,sw]]]]];
Sum[Coefficient[gg,z,j] j! (n-j-1)!,{j,n-1}],{i,n}]/n!]

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In[12]:= Timing[ssPowerPlus[votosUE,62]]
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Out[12]= {1.65 Second, { $\frac{7}{60}$ ,  $\frac{7}{60}$ ,  $\frac{7}{60}$ ,  $\frac{7}{60}$ ,  $\frac{860}{9009}$ ,  $\frac{19883}{360360}$ ,  $\frac{19883}{360360}$ ,  
 $\frac{19883}{360360}$ ,  $\frac{19883}{360360}$ ,  $\frac{743}{16380}$ ,  $\frac{743}{16380}$ ,  $\frac{1588}{45045}$ ,  $\frac{1588}{45045}$ ,  $\frac{1588}{45045}$ ,  $\frac{932}{45045}$ }  
}
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In[13]:= Timing[ssPowerPlus[votosUE,65]]
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Out[13]= {1.65 Second, { $\frac{21733}{180180}$ ,  $\frac{21733}{180180}$ ,  $\frac{21733}{180180}$ ,  $\frac{21733}{180180}$ ,  $\frac{4216}{45045}$ ,  
 $\frac{2039}{36036}$ ,  $\frac{2039}{36036}$ ,  $\frac{2039}{36036}$ ,  $\frac{2039}{36036}$ ,  $\frac{3587}{90090}$ ,  $\frac{3587}{90090}$ ,  $\frac{2987}{90090}$ ,  $\frac{2987}{90090}$ ,  
 $\frac{2987}{90090}$ ,  $\frac{1667}{90090}$ }}
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In[14]:= shaUE=Table[SetPrecision[ssPowerPlus[votosUE,i],3],{i,61,68}];
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In[15]:= countries=Reverse[{"LU","IR","FI","DE","AU","SW","PO",  
"BE","GR","NE","SP","IT","FR","UK","GE"}];
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In[16]:= TableForm[Transpose[{PUE,VUE,  
shaUE[[1]],shaUE[[2]],shaUE[[3]],shaUE[[4]]}],  
TableHeadings->{countries,{"PEOPLE","VOTES",  
"SHA 61","SHA 62","SHA 63","SHA 64 \n"}}]
```

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Out[16]//TableForm=
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	PEOPLE	VOTES	SHA 61	SHA 62	SHA 63	SHA 64
GE	0.219	0.115	0.119	0.117	0.120	0.119
UK	0.157	0.115	0.119	0.117	0.120	0.119
FR	0.156	0.115	0.119	0.117	0.120	0.119
IT	0.155	0.115	0.119	0.117	0.120	0.119
SP	0.106	0.0920	0.0917	0.0955	0.0924	0.0884
NE	0.0414	0.0575	0.0558	0.0552	0.0566	0.0556
GR	0.0281	0.0575	0.0558	0.0552	0.0566	0.0556
BE	0.0273	0.0575	0.0558	0.0552	0.0566	0.0556
PO	0.0268	0.0575	0.0558	0.0552	0.0566	0.0556
SW	0.0236	0.0460	0.0464	0.0454	0.0402	0.0490
AU	0.0215	0.0460	0.0464	0.0454	0.0402	0.0490
DE	0.0141	0.0345	0.0313	0.0353	0.0331	0.0306
FI	0.0137	0.0345	0.0313	0.0353	0.0331	0.0306
IR	0.00966	0.0345	0.0313	0.0353	0.0331	0.0306
LU	0.00109	0.0230	0.0218	0.0207	0.0226	0.0237

```
In[17]:= TableForm[Transpose[{PUE,VUE,  
shaUE[[5]],shaUE[[6]],shaUE[[7]],shaUE[[8]]}],  
TableHeadings->{countries,{"PEOPLE","VOTES",  
"SHA 65","SHA 66","SHA 67","SHA 68 \n"}}]
```

```
Out[17]//TableForm=
```

	PEOPLE	VOTES	SHA 65	SHA 66	SHA 67	SHA 68
GE	0.219	0.115	0.121	0.118	0.115	0.124
UK	0.157	0.115	0.121	0.118	0.115	0.124
FR	0.156	0.115	0.121	0.118	0.115	0.124
IT	0.155	0.115	0.121	0.118	0.115	0.124
SP	0.106	0.0920	0.0936	0.0921	0.0981	0.0911
NE	0.0414	0.0575	0.0566	0.0558	0.0542	0.0550
GR	0.0281	0.0575	0.0566	0.0558	0.0542	0.0550
BE	0.0273	0.0575	0.0566	0.0558	0.0542	0.0550
PO	0.0268	0.0575	0.0566	0.0558	0.0542	0.0550
SW	0.0236	0.0460	0.0398	0.0472	0.0463	0.0374
AU	0.0215	0.0460	0.0398	0.0472	0.0463	0.0374
DE	0.0141	0.0345	0.0332	0.0316	0.0373	0.0321
FI	0.0137	0.0345	0.0332	0.0316	0.0373	0.0321
IR	0.00966	0.0345	0.0332	0.0316	0.0373	0.0321
LU	0.00109	0.0230	0.0185	0.0220	0.0215	0.0237

