Voting Power in the Council of the European Union under the Nice rules

E. Algaba, J.M. Bilbao[*], J.R. Fernández García, J.J. López
Matemática Aplicada II, Escuela Superior de Ingenieros[†]
Camino de los Descubrimientos s/n, 41092 Sevilla, Spain

Abstract

We provide a new method to compute the Banzhaf power index for weighted voting games as well as weighted double and triple majority games. We calculate the Banzhaf index in an exact way by generating functions, with a significant decrease in the computational complexity. Moreover, the Banzhaf indices are calculated for the decision rules approved in the Nice summit meeting, which will be used in the European Union enlarged to 27 countries. We show that the triple majority rules adopted are quasi equivalent to weighted voting or double majority games, since the required population quota to approve a decision does not change the voting power of the countries. Finally, we conclude with some contradictions found in the Treaty of Nice.

1 Introduction

A weighted voting game is defined on a finite set N of players, which can be people, companies, political parties or countries. Each player $i \in N$ has a number of votes $w_i > 0$, so each coalition of players $S \subseteq N$, has the sum of votes of its components $w(S) = \sum_{i \in S} w_i$. Fixed a quota q to take decisions, a coalition S is winning if $w(S) \geq q$, and is losing if $w(S) < q$. As there are exactly two possibilities for each coalition of players, a weighted voting game is modelled with the simple game $v : 2^N \to \{0, 1\}$, defined by

$$v(S) = \begin{cases} 1; & \text{if } w(S) \geq q; \\ 0; & \text{otherwise.} \end{cases}$$

Consequently, a weighted voting game is represented by the following scheme $v = [q; w_1, \ldots, w_n]$. The power of a player is an 'a priori' measure of his/her influencing capacity, based on computing the capacity of each player to participate in winning coalitions. There are two well-known power indices, the Banzhaf index (Banzhaf, 1965) and the Shapley-Shubik index (Shapley & Shubik, 1954). Both of them give a more precise measure of the power of a player that the number of votes assigned to each player.

[*] E-mail: mbilbao@us.es http://www.esi2.us.es/~mbilbao/

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Another aspect to be mentioned in a weighted voting game is the following: How can we measure the power of a player to block a decision? The answer to this question is that the power of a player to block a decision is the same as the one player has in order to approve it. More precisely, the Banzhaf and Shapley-Shubik indices coincide in the vetoing game and the approval one. So, these indices measure both the capacity of a player to adopt a proposal and to block it (see Straffin, 1983; Dubey & Shapley, 1979).

In this work, we focus on computing the Banzhaf index by generating functions. This allows us to reach some conclusions about the power of the countries under the Nice rules. Some related papers to this, in the sense that they make an study of the power indices before and after the enlargement of the European Union can be found in Lane & Berg (1999), Lane & Mæland (2000), and Felsenthal & Machover (2001). In section 2 we recall briefly the concept of generating function to solve the problem of counting those coalitions that have certain properties. Section 3 is devoted to describe the computation of the Banzhaf index using generating functions. The starting point of this approach are in works due to David G. Cantor (see Lucas, 1983), Brams & Affuso (1976), and Tannenbaum (1997). Taking into account that the two decision rules approved in the Nice summit are triple majority voting games, section 4 introduces new algorithms to calculate the Banzhaf index for these games. In section 5, we calculate the Banzhaf index for the Council of the European Union enlarged to 27 countries under the Nice rules.

2 Generating functions

The theory of generating functions provides a method to count the number of elements $c(k)$ of a finite set, when these elements have a determined configuration depending on a variable k. Given a sequence $\{c(k)\}_{k \geq 0}$ its generating function is the formal power series $f(x) = \sum_{k \geq 0} c(k)\, x^k$. For example, if $n \in \mathbb{N}$ is a natural number, then the number of sets with k elements of the set $N = \{1, 2, \ldots, n\}$ is given by the binomial coefficient

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!}.$$

A generating function of the binomial coefficients is

$$(1+x)^n = \sum_{k \geq 0} \binom{n}{k} x^k.$$

From now on, to simplify the notation, $c(k)$ will be written as c_k. We also consider generating functions of several variables, defined by

$$f(x, y, z) = \sum_{k \geq 0} \sum_{j \geq 0} \sum_{l \geq 0} c_{kjl} x^k y^j z^l.$$

3 Banzhaf power index

We first introduce the concept of swing. A swing for player i is a pair of coalitions $(S \cup \{i\}, S)$ such that $i \notin S$; the coalition $S \cup \{i\}$ is winning and S is losing. From now on, we will write $S \cup i$ and $S \setminus i$ instead of $S \cup \{i\}$ and $S \setminus \{i\}$, respectively. For each player $i \in N$, the number of swings for player i in the game $(N; v)$ is denoted by $\eta_i(v)$. This is the number of coalitions for which player i is decisive. The total number of swings is $\eta(v) = \sum_{i \in N} \eta_i(v)$, and the normalized Banzhaf index of player i is given by

$$\beta_i(v) = \frac{\eta_i(v)}{\eta(v)}.$$

Coleman (1973) considered two indices to measure the power to prevent and initiate an action. In the above notation, these two Coleman indices are

$$\gamma_i(v) = \frac{\eta_i(v)}{\omega}; \quad \gamma_i^*(v) = \frac{\eta_i(v)}{\lambda};$$

where ω and λ are the total number of winning and losing coalitions, respectively. For a comprehensive work on the problem of measuring voting power, see Felsenthal & Machover (1998).

Let $v = [q; w_1, \ldots, w_n]$ be a weighted voting game. Then, the number of swings of player i, that is, the total number of coalitions that were losing and become winners when player i is incorporated is given by

$$\eta_i(v) = \sum_{k=q-w_i}^{q-1} b_k^i;$$

where b_k^i is the number of coalitions $S \subseteq N$ such that $i \notin S$ and $w(S) = k$. Brams & Affuso (1976) introduce the following generating function

$$B_i(x) = \prod_{j=1, j\neq i} (1 + x^{w_j}) = \sum_{k=0}^{w(N\setminus i)} b_k^i x^k;$$

that allows to compute these numbers.

Example 1. The weighted voting game corresponding to the Seville Council is given by

$$v = [17; 13, 12, 6, 2].$$

Computing the normalized Banzhaf index, using the definition, requires to determine previously the weight and the value of each coalition.

Coalition	$w(S)$	$v(S)$
$\emptyset$	0	0
$\{1\}$	13	0
$\{2\}$	12	0
$\{3\}$	6	0
$\{4\}$	2	0
$\{1,2\}$	25	1
$\{1,3\}$	19	1
$\{1,4\}$	15	0

Coalition	$w(S)$	$v(S)$
$\{2,3\}$	18	1
$\{2,4\}$	14	0
$\{3,4\}$	8	0
$\{1,2,3\}$	31	1
$\{1,2,4\}$	27	1
$\{1,3,4\}$	21	1
$\{2,3,4\}$	20	1
$\{1,2,3,4\}$	33	1

The coalitions S such that $(S \cup \{i\}, S)$ is a swing for player i, are

Player	Coalitions
1	$\{\{2\},\{3\},\{2,4\},\{3,4\}\}$
2	$\{\{1\},\{3\},\{1,4\},\{3,4\}\}$
3	$\{\{1\},\{2\},\{1,4\},\{2,4\}\}$
4	$\emptyset$

Therefore, the number of swings for each player is

Player	$\eta_i(v)$
1	4
2	4
3	4
4	0

The total number of swings is

$$\bar{\eta}(v) = \sum_{i \in N} \eta_i(v) = 12;$$

and the normalized Banzhaf index is

$$\beta(v) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0 \right).$$

In order to calculate the Banzhaf index by generating functions, we first obtain the functions $B_i(x) = \prod_{j=1, j \neq i} (1 + x^{w_j})$.

$B_1(x) = \left(1 + x^{12}\right)(1 + x^6)(1 + x^2) = 1 + x^2 + x^6 + x^8 + x^{12} + x^{14} + x^{18} + x^{20};$

$B_2(x) = (1 + x^{13})(1 + x^6)(1 + x^2) = 1 + x^2 + x^6 + x^8 + x^{13} + x^{15} + x^{19} + x^{21};$

$B_3(x) = (1 + x^{13})(1 + x^{12})(1 + x^2) = 1 + x^2 + x^{12} + x^{13} + x^{14} + x^{15} + x^{25} + x^{27};$

$B_4(x) = (1 + x^{13})(1 + x^{12})(1 + x^6) = 1 + x^6 + x^{12} + x^{13} + x^{18} + x^{19} + x^{25} + x^{31}.$

The number of swings of player i is obtained by

$$\eta_i(v) = \sum_{k=q-w_i}^{q-1} b_k^i.$$

Therefore,

$$\eta_1(v) = \sum_{k=4}^{16} b_k^1 = 4; \quad \eta_2(v) = \sum_{k=5}^{16} b_k^2 = 4;$$
$$\eta_3(v) = \sum_{k=11}^{16} b_k^3 = 4; \quad \eta_4(v) = \sum_{k=15}^{16} b_k^4 = 0.$$

Finally, the total number of swings is

$$\bar{\eta}(v) = \sum_{i \in N} \eta_i(v) = 12;$$

and the normalized Banzhaf index is

$$\beta(v) = \left(\frac{1}{3}; \frac{1}{3}; \frac{1}{3}; 0 \right).$$

In the previous example, the number of non-zero coefficients of the polynomials $B_i(x)$ is bounded by the non-zero coefficients of the polynomial

$$B(x) = \left(1 + x^{13}\right)\left(1 + x^{12}\right)\left(1 + x^{6}\right)\left(1 + x^{2}\right).$$

In general, for any weighted voting game $v = [q; w_1; \ldots; w_n]$, the number c of non-zero coefficients of the polynomial $B(x)$ satisfies

$$n + 1 \leq c \leq \min(2^n; w(N) + 1).$$

If the number of non-zero coefficients of the polynomial $B(x)$ is known then a bound of the complexity of the problem of computing the normalized Banzhaf index can be obtained. Next, we include three examples to clarify this question.

Example 2. In the weighted voting game $v = [17; 13; 12; 6; 2]$ of the above example, the polynomial $B(x)$ is given by

$$B(x) = \left(1 + x^{13}\right)\left(1 + x^{12}\right)\left(1 + x^{6}\right)\left(1 + x^{2}\right).$$

It is satisfied that $c = 16$; $w(N) + 1 = 34$ and $2^4 = 16$.

Example 3. The composition of the Spanish Parliament, corresponding to the legislature 1996–2000, was the following:

1 (PP) 156 seats	7 (BNG) 2 seats
2 (PSOE) 141 seats	8 (HB) 2 seats
3 (IU) 21 seats	9 (ERC) 1 seat
4 (CiU) 16 seats	10 (EA) 1 seat
5 (PNV) 5 seats	11 (UV) 1 seat
6 (CC) 4 seats	

The power of the political parties in the Parliament can be analyzed by the weighted voting game

$$v = [176;\ 156;141;21;16;5;4;2;2;1;1;1]:$$

The polynomial $B(x)$ is given by

$$\left(1+x^{156}\right)\left(1+x^{141}\right)\left(1+x^{21}\right)\left(1+x^{16}\right)\left(1+x^{5}\right)\left(1+x^{4}\right)\left(1+x^{2}\right)^{2}(1+x)^{3};$$

and it is satisfied that $c = 177$; $w(N)+1 = 351$ and $2^{11} = 2048$.

Example 4. Let us consider the weighted voting game corresponding to the decision-making process in the Council of the European Union. The set of players is formed by the 15 member states:

{Germany, The United Kingdom, France, Italy, Spain, The Netherlands, Greece, Belgium, Portugal, Sweden, Austria, Denmark, Finland, Ireland, Luxembourg}.

With the current votes, this game is represented by

$$v = [62;\ 10;10;10;10;8;5;5;5;5;4;4;3;3;3;2]:$$

Then, the polynomial $B(x)$ is

$$\left(1+x^{10}\right)^{4}\left(1+x^{8}\right)\left(1+x^{5}\right)^{4}\left(1+x^{4}\right)^{2}\left(1+x^{3}\right)^{3}\left(1+x^{2}\right)$$

and it is satisfied that $c = 86$; $w(N)+1 = 88$ and $2^{15} = 32768$.

For computing the Banzhaf index in the weighted voting game corresponding to the Council of the European Union, we need to analyze the coefficients of 15 polynomials $B_i(x)$, one for each player, and the number of non-zero coefficients of each one of these polynomials is bounded by $c = 86$. This number is the reference that we must use to analyze the complexity of the problem using generating functions. In this case, the number c is far smaller than the total number of coalitions 2^{15}. It explains the utility of using generating functions and the speed of the algorithms that implement them. Using Mathematica, the calculation of the Banzhaf indices by generating functions for the countries in the Council of the European Union can be done in less than a second in any personal computer.

In the European Union game, the number $c = 86$ is smaller than the number $c = 177$ obtained for the Parliament game. So, although the European Union has more players than the Parliament, the computation time of the Banzhaf indices, by generating functions, is smaller for the European Union game.

4 Banzhaf index with double and triple majority

One of the essential agreements in the European Union summit, held in Nice in December 2000, has been the approval of new voting systems due to the European Union enlargement. Several voting systems have been analyzed to regulate the decision making in the Council of the European Union, being adopted two triple majority models with a weighting of the current votes.

A weighted double majority game is the composition of two weighted voting games. Let us consider two weighted voting games $v_1 = [q; w_1; \ldots; w_n]$ and $v_2 = [p; p_1; \ldots; p_n]$. The weighted double majority game, denoted by $v_1 \wedge v_2$, is defined by

$$(v_1 \wedge v_2)(S) = \begin{cases} 1; & \text{if } w(S) \geq q \text{ and } p(S) \geq p; \\ 0; & \text{otherwise.} \end{cases}$$

Similarly, a weighted triple majority game can be interpreted as the composition of three weighted voting games $v_t = \left[q^t; w_1^t; \ldots; w_n^t\right]$, with $t = 1; 2; 3$. Its characteristic function, denoted by $v_1 \wedge v_2 \wedge v_3$ satisfies, for all $S \subseteq N$;

$$(v_1 \wedge v_2 \wedge v_3)(S) = \begin{cases} 1; & \text{if } w^t(S) \geq q^t \text{ for all } t = 1; 2; 3; \\ 0; & \text{otherwise,} \end{cases}$$

where

$$w^t(S) = \sum_{i \in S} w_i^t.$$

These majority games are simple games, and hence the Banzhaf index is defined in the same way,

$$\beta_i(v) = \frac{\eta_i(v)}{\eta(v)}.$$

Consequently, we need to compute for every player i, its number of swings. This number can be calculated from the set of winning coalitions in which player i is and, inside of this set, the subset of coalitions where the presence of player i is not necessary for winning. For a weighted double majority game, the number of swings of player i is given by formula

$$\eta_i(v) = \sum_{k=q-w_i}^{w(N \setminus i)} \sum_{r=p-p_i}^{p(N \setminus i)} b_{kr}^i - \sum_{k=q}^{w(N \setminus i)} \sum_{r=p}^{p(N \setminus i)} b_{kr}^i;$$

where b_{kr}^i is the number of coalitions S that do not include player i such that $w(S) = k$ and $p(S) = r$ (see Fernández García, 2000a): In this case, the computation of the numbers $\left(b_{kr}^i\right)_{k;r \geq 0}$, for every player $i \in N$, can be done by the following generating function

$$B_i(x; y) = \prod_{j=1; j \neq i} (1 + x^{w_j} y^{p_j}) = \sum_{k=0}^{w(N \setminus i)} \sum_{r=0}^{p(N \setminus i)} b_{kr}^i x^k y^r.$$

The number c of non-zero coefficients of $B(x;y) = \prod_{j=1}^{n}(1 + x^{w_j} y^{p_j})$ satisfies

$$n + 1 \leq c \leq \min(2^n; w(N)\,p(N) + 1);$$

which provides us a measure of the complexity of the problem. The computational complexity of these algorithms has been studied by Fernández García et al. (2000b).

Example 5. Suppose that for a type of agreements at the Seville Council, we must count on the support of the absolute majority of his members; which, also, have to represent the absolute majority of their councillors. It can be described by the weighted double majority game given by $v = v_1 \wedge v_2$, where $v_1 = [17; 13; 12; 6; 2]$ and $v_2 = [3; 1; 1; 1; 1]$. Its characteristic function is

$$(v_1 \wedge v_2)(S) = \begin{cases} 1; & \text{if } w(S) \geq 17 \text{ and } p(S) \geq 3 \\ 0; & \text{otherwise.} \end{cases}$$

We first calculate the functions $B_i(x;y) = \prod_{j=1;\, j\neq i}^{n}(1 + x^{w_j} y^{p_j})$.

$$B_1(x;y) = 1 + x^2y + x^6y + x^8y^2 + x^{12}y + x^{14}y^2 + x^{18}y^2 + x^{20}y^3;$$
$$B_2(x;y) = 1 + x^2y + x^6y + +x^{13}y + x^8y^2 + x^{15}y^2 + x^{19}y^2 + x^{21}y^3;$$
$$B_3(x;y) = 1 + x^2y + x^{12}y + x^{13}y + x^{14}y^2 + x^{15}y^2 + x^{25}y^2 + x^{27}y^3;$$
$$B_4(x;y) = 1 + x^6y + x^{12}y + x^{13}y + x^{18}y^2 + x^{19}y^2 + x^{25}y^2 + x^{31}y^3.$$

The following difference is calculated in order to compute the swings for every player

$$\eta_i(v) = \sum_{k=q-w_i}^{w(N\backslash i)} \sum_{r=p-p_i}^{p(N\backslash i)} b^i_{kr} - \sum_{k=q}^{w(N\backslash i)} \sum_{r=p}^{p(N\backslash i)} b^i_{kr};$$

so, the swings are given by

$$\eta_1(v) = \sum_{k=4}^{20} \sum_{r=2}^{3} b^1_{kr} - \sum_{k=17}^{20} \sum_{r=3}^{3} b^1_{kr} = 4 - 1 = 3;$$
$$\eta_2(v) = \sum_{k=5}^{21} \sum_{r=2}^{3} b^2_{kr} - \sum_{k=17}^{21} \sum_{r=3}^{3} b^2_{kr} = 4 - 1 = 3;$$
$$\eta_3(v) = \sum_{k=11}^{27} \sum_{r=2}^{3} b^3_{kr} - \sum_{k=17}^{27} \sum_{r=3}^{3} b^3_{kr} = 4 - 1 = 3;$$
$$\eta_4(v) = \sum_{k=15}^{31} \sum_{r=2}^{3} b^4_{kr} - \sum_{k=17}^{31} \sum_{r=3}^{3} b^4_{kr} = 4 - 1 = 3.$$

As the total number of swings is

$$\bar{\eta}(v) = \sum_{i=1}^{4} \eta_i(v) = 12;$$

the normalized Banzhaf index is

$$\beta(v) = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right).$$

For a weighted triple majority game the number of swings of player i is

$$\eta_i(v) = \sum_{k_1=q^1-w_i^1}^{w^1(N\setminus i)} \sum_{k_2=q^2-w_i^2}^{w^2(N\setminus i)} \sum_{k_3=q^3-w_i^3}^{w^3(N\setminus i)} b^i_{k_1k_2k_3} - \sum_{k_1=q^1}^{w^1(N\setminus i)} \sum_{k_2=q^2}^{w^2(N\setminus i)} \sum_{k_3=q^3}^{w^3(N\setminus i)} b^i_{k_1k_2k_3},$$

where $b^i_{k_1k_2k_3}$ is the number of coalitions S that do not include player i such that $w^t(S) = k_t$, $t = 1, 2, 3$. In this case, we can obtain the numbers $\left\{b^i_{k_1k_2k_3}\right\}_{k_t \geq 0}$ for each player $i \in N$, from the generating function

$$B_i(x, y, z) = \prod_{j=1;\, j\neq i} \left(1 + x^{w_j^1} y^{w_j^2} z^{w_j^3}\right)$$

$$= \sum_{k_1=0}^{w^1(N\setminus i)} \sum_{k_2=0}^{w^2(N\setminus i)} \sum_{k_3=0}^{w^3(N\setminus i)} b^i_{k_1k_2k_3} x^{k_1} y^{k_2} z^{k_3}.$$

The number of non-zero coefficients c of the polynomial

$$B(x, y, z) = \prod_{j=1} \left(1 + x^{w_j^1} y^{w_j^2} z^{w_j^3}\right)$$

is bounded in the following way

$$n + 1 \leq c \leq \min\left(2^n, \prod_{t=1}^{3} w^t(N) + 1\right).$$

Computing the normalized Banzhaf index of n players in a weighted triple majority game requires a running time bounded by $n^2 c$ where c is the number of non-zero coefficients of the generating function $B(x, y, z)$.

5 Nice rules for the European Union enlargement

The Journal of Theoretical Politics 11 (1999) has recently published several articles about the modelling of decision making process in the European Union (EU). Garrett & Tsebelis (1999a, 1999b) criticized the classical voting power method in the context of the EU because the power indices do not take into account the preferences of players and the institutional structure of the EU. Lane & Berg (1999) assert that:

> "Cooperative theory solution concepts, on the other hand, assume that players have specified preferences and can make the types of binding commitments typically required to cement together a particular coalition in support of a particular outcome. Cooperative

> solutions, including power indices, therefore, are directly applicable to policy analysis only in conjunction with assumptions about preferences (spatial or otherwise) and in circumstances that suggest that binding agreements may be feasible. Therefore, criticism of cooperative game theory and power indices may be misplaced insofar as constitutional analysis is concerned, but may have more weight insofar as policy analysis is concerned, although formal power indices often help explain voting outcomes."

Holler & Widgrén (1999) propose some ideas to combine spatial voting games and power index models. Steunenberg, Schmidtchen & Koboldt (1999) define the strategic power index, a new approach which is based on spatial and sequential models of decision making.

We agree with Lane & Berg on the need for a priori measures of power. However, we believe that the best choice is a method that takes the voting power theory, so-called intergovernmentalism and supplements it by the institutional analysis of the EU legislative process.

The Council of Ministers of the EU represents the national governments of the member states. The Council uses a voting system of qualified majority to pass new legislation. The Nice European Council in December 2000 established two decision rules for the EU enlarged to 27 countries. These rules are contained in the Declaration on the enlargement of the European Union and the Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union (Official Journal of the European Communities 10.3.2001, C 80/80-85).

Felsenthal & Machover (2001) analyzed in terms of a priori measures of power these decision rules for the Council of Ministers of the EU. They used the Bräuninger-König IOP 1.0 program and the Lemma 3.3.12 in Felsenthal & Machover (1998) to calculate the voting power of each one of the present 15 members and the future 27 ones.

The new version of the program IOP 2.0 allows us to calculate voting power indices for the post Nice institutions of the EU where Council members have two kinds of weighted and one unweighted vote. In addition, an option for reporting winning and minimal winning coalitions is implemented (see Bräuninger & König, 2001).

The normative question of what the voting weights should be in order that the decision rules are fair is considered by Leech (2001). He reached the following main conclusions:

> "(1) that the weights laid down in the Nice Treaty are close to being fair, the only significant discrepancies being the under representation of Germany and Romania, and the over representation of Spain and Poland; (2) the majority quota required for a decision is set too high for the Council of Ministers to be an effective decision making body."

We next present our results concerning to the Banzhaf index using the algorithms of the previous sections. We compute these indices under the two decision rules prescribed by the Treaty of Nice. Each member state represented in the future Council is considered an individual player. The players in the Council of the EU enlarged to 27 countries are:

{Germany, The United Kingdom, France, Italy, Spain, Poland, Romania, The Netherlands, Greece, Czech Republic, Belgium, Hungary, Portugal, Sweden, Bulgaria, Austria, Slovak Republic, Denmark, Finland, Ireland, Lithuania, Latvia, Slovenia, Estonia, Cyprus, Luxembourg, Malta}.

The …rst decision rule is the weighted triple majority game $v_1 \wedge v_2 \wedge v_3$, where the three weighted voting games corresponding to votes, countries and population, are the following:

v_1 = [255; 29,29,29,29,27,27,14,13,12,12,12,12,12,10,10,10,7,7,7,7,7,4,4,4,4,4,3];

v_2 = [14; 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];

v_3 = [620; 170,123,122,120,82,80,47,33,22,21,21,21,21,18,17,17,11,11,11,8,8,5,4,3,2,1,1]:

The game v_3 is de…ned assigning to every country a number of votes equal to the rate per thousand of its population over the total population and the quota represents 62% of the total population. So, a voting will be favorable if it counts on the support of 14 countries with at least 255 votes, and with at least 62% of the total population.

The second decision rule is the weighted triple majority game $v_1 \wedge v_2^0 \wedge v_3$; where the weighted voting game v_2^0 consists of a quali…ed majority of 2=3 of the countries, that is,

v_2^0 = [18; 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1]:

Next, we give a table which contains the Banzhaf power indices of the countries if the …rst decision rule is used. The percentage of population over the total is included in the column called Pop. I. The percentage of votes of every country over the total is shown on the column Vote I. The Banzhaf indices of the game v_1 are included in the column Game1, and the Banzhaf indices of the weighted triple majority game $v_1 \wedge v_2 \wedge v_3$ are in the column Game3a.

Countries	Population	Votes	Pop. I	Votes I	Game1	Game3a
Germany	82.038	29	0.170	0.084	0.0778	0.0778
The United Kingdom	59.247	29	0.123	0.084	0.0778	0.0778
France	58.966	29	0.123	0.084	0.0778	0.0778
Italy	57.612	29	0.120	0.084	0.0778	0.0778
Spain	39.394	27	0.082	0.078	0.0742	0.0742
Poland	38.667	27	0.080	0.078	0.0742	0.0742
Romania	22.489	14	0.047	0.041	0.0426	0.0426
The Netherlands	15.760	13	0.033	0.038	0.0397	0.0397
Greece	10.533	12	0.022	0.035	0.0368	0.0368
Czech Republic	10.290	12	0.021	0.035	0.0368	0.0368
Belgium	10.213	12	0.021	0.035	0.0368	0.0368
Hungary	10.092	12	0.021	0.035	0.0368	0.0368
Portugal	9.980	12	0.021	0.035	0.0368	0.0368
Sweden	8.854	10	0.018	0.029	0.0309	0.0309
Bulgaria	8.230	10	0.017	0.029	0.0309	0.0309
Austria	8.082	10	0.017	0.029	0.0309	0.0309
Slovak Republic	5.393	7	0.011	0.020	0.0218	0.0218
Denmark	5.313	7	0.011	0.020	0.0218	0.0218
Finland	5.160	7	0.011	0.020	0.0218	0.0218
Ireland	3.744	7	0.008	0.020	0.0218	0.0218
Lithuania	3.701	7	0.008	0.020	0.0218	0.0218
Latvia	2.439	4	0.005	0.012	0.0125	0.0125
Slovenia	1.978	4	0.004	0.012	0.0125	0.0125
Estonia	1.446	4	0.003	0.012	0.0125	0.0125
Cyprus	0.752	4	0.002	0.012	0.0125	0.0125
Luxembourg	0.429	4	0.001	0.012	0.0125	0.0125
Malta	0.379	3	0.001	0.009	0.0094	0.0094

Table 1: The Banzhaf index under the first rule

From table 1, we can deduce the following conclusions:

- The first decision rule, consisting of a triple majority system, is quasi equivalent to the first game of qualified majority. The power of all countries is almost the same as the power with the simple game v_1, with the double game $v_1 \wedge v_2$, and with the triple one $v_1 \wedge v_2 \wedge v_3$.

- The second decision rule, that differs from the first one because it requires the approval of at least $2/3$ of the countries, is quasi equivalent to the weighted double majority game $v_1 \wedge v_2^{0}$. In this rule, the required population quota to take a decision does not change the power of the countries.

- The strategy of France is to make a balance of power in the Council of the EU, since it has the same power as Germany, with the two decision rules adopted.

- Germany, The United Kingdom, France and Italy have a power index equal to 0:0778, which is obviously inferior to its respective population indices.
- The two decision rules penalize Germany, whenever its power index (0:0778) is compared with its population index (0:170).
- The position of Spain is very balanced: its Banzhaf index is 0:0742, its vote index 0:078 and its population index 0:082. The position of Poland is similar to Spain.
- Romania has a vote index and a power one inferior to its population index. The opposite occurs to The Netherlands: its vote index and its power one are superior to its population index.
- The rest of countries has a power index superior to its population and vote indices. With respect to this, Luxembourg is the country with the best position: its Banzhaf index is 0:0125, and its population index 0:0009.

In a game with 27 players, the total number of coalitions which can be formed is

$$2^{27} = 134217728:$$

For this reason, the argumentations 'ad hoc' based only on the analysis from half a dozen of winning coalitions, lack rational foundation.

For example, for the game v_1, with $q = 255$, which is called Game1, the total number of swings is 28186428; and for the weighted triple majority game $v_1 \wedge v_2 \wedge v_3$ (votes, majority of countries and more than 62% of the population) which is named Game3a, the number of swings is 28186324. The di¤erence is insigni…cant, only 104 swings over a total superior to 28 millions.

The following table shows the number of swings for every country in both games.

Countries	Game 1	Game 3a	Di¤erence
Germany	2193664	2193654	¡ 10
The United Kingdom	2193664	2193650	¡ 14
France	2193664	2193650	¡ 14
Italy	2193664	2193650	¡ 14
Spain	2091380	2091358	¡ 22
Poland	2091380	2091358	¡ 22
Romania	1200504	1200482	¡ 22
The Netherlands	1120138	1120116	¡ 22
Greece	1038492	1038476	¡ 16
Czech Republic	1038492	1038476	¡ 16
Belgium	1038492	1038476	¡ 16
Hungary	1038492	1038476	¡ 16
Portugal	1038492	1038476	¡ 16
Sweden	871654	871654	0
Bulgaria	871654	871654	0
Austria	871654	871654	0
Slovak Republic	614702	614712	10
Denmark	614702	614712	10
Finland	614702	614712	10
Ireland	614702	614712	10
Lithuania	614702	614712	10
Latvia	352374	352384	10
Slovenia	352374	352384	10
Estonia	352374	352384	10
Cyprus	352374	352384	10
Luxembourg	352374	352384	10
Malta	265568	265584	16

Table 2: Number of swings

Germany has as much power as The United Kingdom, France and Italy. In the weighted triple majority game $v_1 ^\wedge v_2 ^\wedge v_3$ the di¤erence is only 4 swings with respect to 28 millions. Concerning the weighted triple majority game $v_1 ^\wedge v_2^0 ^\wedge v_3$, the di¤erence is also 4 swings in favor of Germany, over 24 millions and a half of swings. Consequently, the di¤erences between the Banzhaf indices of Germany and The United Kingdom, France and Italy are, respectively, smaller than 1:4 £ $10^{i\,7}$ and 1:6 £ $10^{i\,7}$.

Next, we include the computations corresponding to the Banzhaf index for the second decision rule, i.e., for the game $v_1 ^\wedge v_2^0 ^\wedge v_3$, labeled Game 3b. In a similar way, in both cases, the results corresponding to the games v_1 labeled Game 1, and $v_1 ^\wedge v_2^0$ labeled Game 2b, are compared. The conclusion, just as we anticipated before, is that the results corresponding to the games $v_1 ^\wedge v_2^0 ^\wedge v_3$ and $v_1 ^\wedge v_2^0$ are almost the same.

Countries	Game 1	Game 2b	Game 3b
Germany	0.0778	0.0665	0.0665
The United Kingdom	0.0778	0.0665	0.0665
France	0.0778	0.0665	0.0665
Italy	0.0778	0.0665	0.0665
Spain	0.0742	0.0631	0.0631
Poland	0.0742	0.0631	0.0631
Romania	0.0426	0.0407	0.0407
The Netherlands	0.0397	0.0386	0.0386
Greece	0.0368	0.0366	0.0366
Czech Republic	0.0368	0.0366	0.0366
Belgium	0.0368	0.0366	0.0366
Hungary	0.0368	0.0366	0.0366
Portugal	0.0368	0.0366	0.0366
Sweden	0.0309	0.0325	0.0325
Bulgaria	0.0309	0.0325	0.0325
Austria	0.0309	0.0325	0.0325
Slovak Republic	0.0218	0.0263	0.0263
Denmark	0.0218	0.0263	0.0263
Finland	0.0218	0.0263	0.0263
Ireland	0.0218	0.0263	0.0263
Lithuania	0.0218	0.0263	0.0263
Latvia	0.0125	0.0198	0.0198
Slovenia	0.0125	0.0198	0.0198
Estonia	0.0125	0.0198	0.0198
Cyprus	0.0125	0.0198	0.0198
Luxembourg	0.0125	0.0198	0.0198
Malta	0.0094	0.0177	0.0177

Table 3: The Banzhaf index under the second rule

The algorithms and the power indices obtained for the EU15, taking into account the Nice weighting of votes, are contained in the notebook of the system Mathematica, due to Bilbao et al. (2001a). The results corresponding to the decision rules approved in Nice, for the EU enlarged to 27 countries, can be found in Bilbao et al. (2001b).

6 Contradictions in the Treaty of Nice

The Treaty of Nice contains the Declaration on the enlargement of the European Union and the Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union, which includes agreements that are a contradiction in terms. So, on pages 82–83 of the Treaty after table 2, which establishes the weighting of votes in the Council of 27 countries, is written textually:

> "Acts of the Council shall require for their adoption at least 258 votes in favour, cast by a majority of members, where this Treaty requires them to be adopted on a proposal from the Commission. In other cases, for their adoption acts of the Council shall require at least 258 votes in favour cast by at least two-thirds of the members. When a decision is to be adopted by the Council by a qualified majority, a member of the Council may request verification that the Member States constituting the qualified majority represent at least 62% of the total population of the Union. If that condition is shown not to have been met, the decision in question shall not be adopted."

Consequently, the qualified majority of 258 votes over a total of 345 votes represents a percentage of 74.78%. On the other hand, on page 85 of the Treaty, the Declaration on the qualified majority threshold establishes that:

> "Insofar as all the candidate countries listed in the Declaration on the enlargement of the European Union have not yet acceded to the Union when the new vote weightings take effect (1 January 2005), the threshold for a qualified majority will move, according to the pace of accessions, from a percentage below the current one to a maximum of 73.4%. When all the candidate countries mentioned above have acceded, the blocking minority, in a Union of 27, will be raised to 91 votes, and the qualified majority threshold resulting from the table given in the Declaration on enlargement of the European Union will be automatically adjusted accordingly."

When all candidate states become member states of the EU, the total of votes in the Council will be $n = 345$. Therefore, a blocking minority of $b = 91$ votes will automatically imply a qualified majority of $q = n + 1 - b = 255$ votes, which corresponds to 73.91% of the votes. As a result:

- The two qualified majorities of 258 and 255 votes, adopted in the Treaty of Nice for the Council of 27 countries, are superior to the maximum percentage of 73.4%, fixed in the Declaration on the qualified majority threshold.

- The qualified majority of 258 votes will never be applied. The reason is that the inclusion of all the candidates implies a qualified majority of 255 votes. In the case that all adhesions do not take place, the majority threshold has to be smaller or equal to 73.4% from a maximum of 342 votes (345 minus 3), that is, less or equal than 251 votes.

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