EUROPEAN CONVENTION *VERSUS* NICE TREATY

E. ALGABA, J. M. BILBAO, J.R. FERNÁNDEZ

Abstract. The aim of this paper is to analyze the distribution of power in the Council of the enlarged European Union. By using generating funcions, we calculate the Banzhaf power indices for the european countries in the Council of Ministers under the decision rules prescribed by the Treaty of Nice and the new rules proposed by the European Convention. Moreover, we analyze the power of the European citizens under the egalitarian model proposed by Felsenthal and Machover [6].

Key words. Power indices, Banzhaf index, European Union

AMS subject classifications. 91A12

1. Introduction

The power of a country in a supranational organization like the International Monetary Fund or the Council of Ministers of the European Union is a numerical measure of its capacity to decide the approval of a motion. This decisive character is measured calculating the number of times that the vote of a country converts a coalition that does not reached the quota to take decisions in a winning coalition. The power indices are *a priori* measures of this power, the most useful are the Shapley-Shubik [8] and Banzhaf [2] indices. Both of them provide a much more exact measure of the power of a player than the number of votes that he/she is entitled to cast. Another planned question in the decision-making is the following: How the power of a country is measured to block a decision? The answer to this question is that the power of a country to block decisions is the same that it has to approve them. In other words, both the Banzhaf index and Shapley-Shubik index coincide in block and approval situations. Thus, these indices measure both the capacity of a country to approve a proposal and block it.

One of the fundamental agreements of the Intergovernmental Conference of the European Union, which took place in Nice in December 2000, was the approval of new voting systems in order to improve the decision rules for the enlargement of the European Union. Several voting systems were discussed to take decisions in the Council of the European Union, where two models of triple majority with a new weighting of the votes were approved. These models correspond to weighted votes, number of countries and population.

The Nice rules were established to enlarge the European Union to 25 countries, so the total number of coalitions is bigger than 33 millions. For that, the line of reasoning only based on the analysis of a fewness winning coalitions is not a rational method. For example, for the first rule approved in Nice, Germany is decisive in more than nine hundred thousand coalitions.

The new voting rule proposed by the European Convention for the future European Constitution changes in a very remarkable way the power of the countries in the Council. The reason is that the weighted votes, that were approved in Nice are removed and a coalition only need 13 votes, which at least sum up by 60% of the population to approve a decision with the new rule.

2. The power of the European countries

The Council of Ministers of the EU represents the national governments of the member states. The Council uses a voting system of qualified majority to pass new legislation. The Nice European Council in December 2000 established two decision rules for the EU enlarged to 27 countries. These rules are contained in the *Declaration on the enlargement of the European Union* and the *Declaration on the qualified majority threshold and the number of votes for a blocking minority in an enlarged Union* (Official Journal of the European Communities 10.3.2001, C 80/80-85).

Felsenthal and Machover [7] analyzed in terms of a *priori* measures of power these decision rules for the Council of Ministers of the EU. They used the Bräuninger-König IOP 1.0 program and the Lemma 3.3.12 in Felsenthal and Machover [6] to calculate the voting power of each one of the present 15 members and the future 27 ones. The new version of the program IOP 2.0 allows us to calculate voting power indices for the post Nice institutions of the EU where Council members have two kinds of weighted and one unweighted vote. In addition, an option for reporting winning and minimal winning coalitions is implemented (see Bräuninger and König [4]).

Definition 2.1. *A simple game is a pair (N, v) where $N = \{1, \dots, n\}$ is the set of players and $v : 2^N \to \{0, 1\}$ is the characteristic function which satisfies $v(\emptyset) = 0$ and $v(S) \leq v(T)$ whenever $S \subseteq T$. A coalition is winning if $v(S) = 1$, and losing if $v(S) = 0$.*

We introduce a special class of simple games called *weighted voting games*. The symbol $[q;\ w_1, \dots, w_n]$ will be used, where q and $w_1, \dots, w_n$ are positive integers with

$$w_i < q \leq \sum_{i=1}^{n} w_i,$$

for $i = 1, \dots, n$. Here there are n players, w_i is the number of votes of player i, and q is the quota needed for a coalition to win.

Then, the above symbol represents the simple game (N, v) defined by

$$v(S) = \begin{cases} 1, & \text{if } w(S) \geq q, \\ 0, & \text{if } w(S) < q, \end{cases}$$

where $S \subseteq N$ and $w(S) = \sum_{i \in S} w_i$.

Given the simple games $(N, v_1), \ldots, (N, v_m)$ we consider the simple game $(N, v_1 \wedge \ldots \wedge v_m)$ defined by

$$(v_1 \wedge \ldots \wedge v_m)(S) = \min \{v_t(S) : 1 \leq t \leq m\}.$$

A *weighted m-majority game* is the simple game $(N, v_1 \wedge \ldots \wedge v_m)$ where the games (N, v_t) are the weighted voting games represented by

$$\left[q^t;\ w_1^t, \ldots, w_n^t\right]$$

for $1 \leq t \leq m$. Then, its characteristic function is given by

$$(v_1 \wedge \ldots \wedge v_m)(S) = \begin{cases} 1, & \text{if } w^t(S) \geq q^t,\ 1 \leq t \leq m, \\ 0, & \text{otherwise,} \end{cases}$$

where $w^t(S) = \sum_{i \in S} w_i^t$. If $m = 2$ or $m = 3$ then we obtain weighted double or triple majority games respectively.

The Banzhaf index is concerned with the number of times each player could change a coalition from losing to winning and it requires to know the number of swings for every player i (see Dubey and Shapley [5]). A *swing* for player i is a pair of coalitions $(S \cup \{i\}, S)$ such that $S \cup \{i\}$ is winning and S is not. For each $i \in N$, we denote by $b_i(v)$ the number of swings for i in game v, that is, the number of winning coalitions in which player i is critical. The total number of swings is

$$\overline{b}(v) = \sum_{i \in N} b_i(v).$$

Definition 2.2. *The normalized Banzhaf index is the vector $\beta(v) \in \mathbb{R}^n$ where*

$$\beta_i(v) = \frac{b_i(v)}{\overline{b}(v)}, \quad 1 \leq i \leq n.$$

The Banzhaf power index depends on the number of ways in which each voter can effect a swing. If there are n players in a voting situation, then the function which measures the worst case running time for computing these indices is in $O(n2^n)$.

We use a combinatorial method based on *generating functions* given by Bilbao et al [1, 3] to calculate the normalized Banzhaf index in pseudo-polynomial time. With this method we obtain the Banzhaf power indices efficiently in the weighted triple and double majority games prescribed by the Treaty of Nice and proposed by the European Convention respectively.

The Council of Ministers is the most important decision-making body of the European Union. The players in the Council of the EU enlarged to 25 countries are:

{Germany, United Kingdom, France, Italy, Spain, Poland, The Netherlands, Greece, Czech Republic, Belgium, Hungary, Portugal, Sweden, Austria, Slovak Republic, Denmark, Finland, Ireland, Lithuania, Latvia, Slovenia, Estonia, Cyprus, Luxembourg, Malta}.

The Nice rule is the weighted triple majority game $v_1 \wedge v_2 \wedge v_3$, where the three weighted voting games corresponding to votes, countries and population, are the following:

$$v_1 = [232;\ 29,29,29,29,27,27,13,12,12,12,12,12,10,10,7,7,7,7,7,4,4,4,4,4,3],$$

$$v_2 = [13;\ 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,],$$

$$v_3 = [620;\ 182,131,131,128,89,86,35,23,23,23,22,22,20,18,12,12,11,8,8,5,4,3,2,1,1].$$

The game v_3 is defined assigning to every country, a number of votes equal to the rate per thousand of its population over the total population and the quota represents the 62% of the total population. So, a voting will be favorable if it counts on the support of 13 countries with at least 232 votes, and with at least the 62% of the population.

The European Convention rule is the weighted double majority game $v_2 \wedge v_3'$, where the quota of the game v_3' is the 60% of the total population, that is,

$$v_3' = [600;\ 182,131,131,128,89,86,35,23,23,23,22,22,20,18,12,12,11,8,8,5,4,3,2,1,1].$$

Next, we present in Table 1 the normalized Banzhaf indices of the European countries. In the column called Population is included the index of population over the total. The normalized Banzhaf indices corresponding to the Nice rule and the European Convention rules with at least the 60% and 66% of the population are in the columns Nice rule, 13&60% and 13&66% respectively.

Countries	Population	Nice rule	13&60%	13&66%
Germany	0.182	0.085	0.133	0.144
United Kingdom	0.131	0.085	0.095	0.108
France	0.131	0.085	0.095	0.108
Italy	0.128	0.085	0.093	0.106
Spain	0.090	0.081	0.069	0.073
Poland	0.086	0.081	0.068	0.070
The Netherlands	0.035	0.042	0.036	0.039
Greece	0.023	0.039	0.029	0.028
Czech Republic	0.023	0.039	0.029	0.028
Belgium	0.023	0.039	0.029	0.028
Hungary	0.022	0.039	0.029	0.028
Portugal	0.021	0.039	0.029	0.028
Sweden	0.020	0.033	0.027	0.026
Austria	0.018	0.033	0.026	0.024
Slovak Republic	0.012	0.023	0.023	0.020
Denmark	0.012	0.023	0.023	0.020
Finland	0.011	0.023	0.022	0.019
Ireland	0.008	0.023	0.021	0.016
Lithuania	0.008	0.023	0.021	0.016
Latvia	0.005	0.013	0.019	0.014
Slovenia	0.004	0.013	0.018	0.013
Estonia	0.003	0.013	0.018	0.012
Cyprus	0.002	0.013	0.017	0.011
Luxembourg	0.001	0.013	0.016	0.011
Malta	0.001	0.001	0.016	0.011

Table 1

The normalized Banzhaf indices are obtained dividing the decisive coalitions of each country by the total number of these coalitions, for that reason Germany increases in power by 50%, the United Kingdom, France and Italy increase by 12% whereas Spain and the rest of European countries, except for the six with smaller population, decrease in power. If we increase the population's quota from 60% to 66%, the results of Table 1 indicate that this imbalance of power stays.

Notice that the consequence of increasing the population's quota from 60% to 66% to take decisions in the Council is an additional increment in Germany's power by 8% and the United Kingdom, France and Italy by 13%. The power of Spain, Poland and the Netherlands increase by 6%, 3% and 8% respectively, but the power of the 19 remaining countries decreases.

Figure 1 shows a three-dimensional graph with the data in Table 1 in order to display the mentioned losses and winnings of power of the countries of the Union European with respect to the Nice rule.

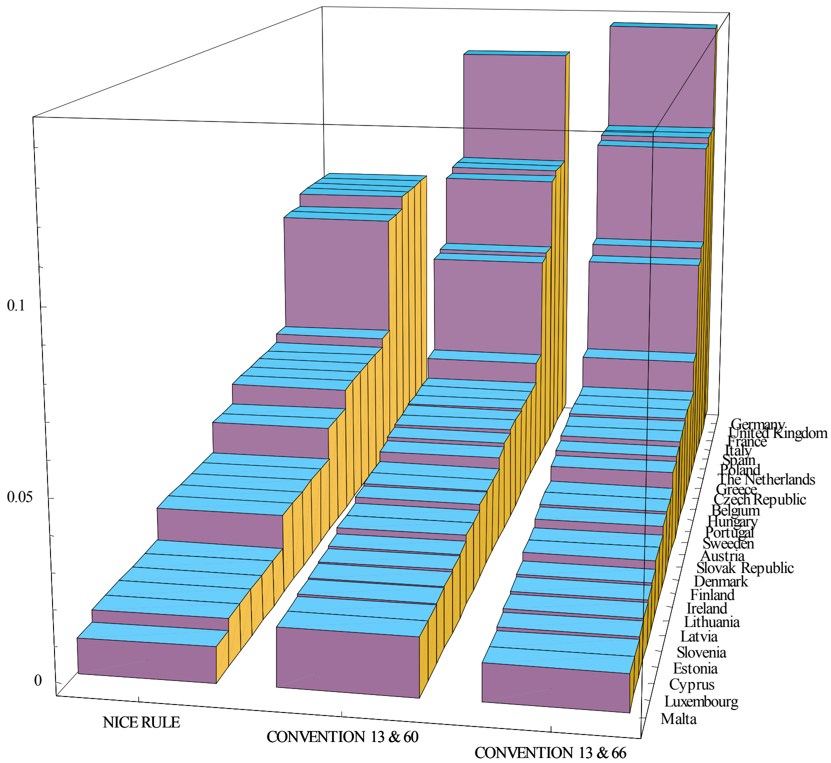

Figure 1

3. The power of the European citizens

The Banzhaf power index measures the power of each country in the Council of the European Union when it takes decisions using weighted majority rules. If we want to measure the power of each European citizen, we should take into account that the citizens in the institutions of the European Union have doubly indirect participation. In a first phase, we must choose our representatives for the European Parliament and for each national parliament. Later, the national institutions designate delegates of each country for the Commission and the Council.

The standard method to analyze the power of a single voter is the following. We consider a weighted majority game (N, v) in which all of the n weights are 1 and the quota is $q = \left\lceil \frac{n+1}{2} \right\rceil$. In this game a voter i is a swing in a winning coalition S if and only if $i \in S$ and $|S| = q$. Then the number

$b_i(v)$ of times that i is a swing is equal to

$$\binom{n-1}{q-1} = \frac{(n-1)!}{(q-1)!\,(n-q)!} = \begin{cases} \dfrac{(2k-1)!}{k!\,(k-1)!} & \text{if } n = 2k, \\ \dfrac{(2k)!}{(k!)^2} & \text{if } n = 2k+1. \end{cases}$$

Using Stirling's approximation formula

$$n! \sim \sqrt{2\pi n}\, e^{-n} n^n,$$

we obtain

$$\frac{(n-1)!}{(q-1)!\,(n-q)!} \sim \begin{cases} \sqrt{\dfrac{2}{\pi n}}\, 2^{n-1} & \text{if } n = 2k, \\ \sqrt{\dfrac{2}{\pi\,(n-1)}}\, 2^{n-1} & \text{if } n = 2k+1. \end{cases}$$

Thus, the swing probabilities of voter i is the number

$$\beta_i'(v) = \frac{\binom{n-1}{q-1}}{2^{n-1}} \sim \sqrt{\frac{2}{\pi n}},$$

if n is sufficiently large.

Definition 3.1. *The probabilistic Banzhaf index is the vector*

$$\beta'(v) = \frac{1}{2^{n-1}}\left(b_1(v), \ldots, b_n(v)\right).$$

This vector is named the Banzhaf measure of voting power by Felsenthal and Machover [6, page 39]. These authors showed that

$$\beta_x' \sim \beta_i' \sqrt{\frac{2}{\pi n_i}},$$

and obtained the following result for the model of two-tier voting system [6, page 66].

Theorem 3.1. *The probabilistic Banzhaf indices β_x' are equal for all voters if and only if the probabilistic Banzhaf indices β_i' of the delegates are proportional to the respective $\sqrt{n_i}$.*

In the model proposed by Felsenthal and Machover [6], if the probabilistic Banzhaf index of the voting system is proportional to the square root of the population of its country then the power of a European citizen is egalitarian.

In Figure 2, which is carried out with the Nice rule, is represented the root square of the population in the horizontal axis, and the Banzhaf index in the vertical axis. Notice that the points in which the power is egalitarian are on the straight line. We can observed that Germany (the located point more to the right) is placed below the straight line; France, the United Kingdom and Italy are on the straight line; whereas Spain and Poland have more power since they are located above the straight line.

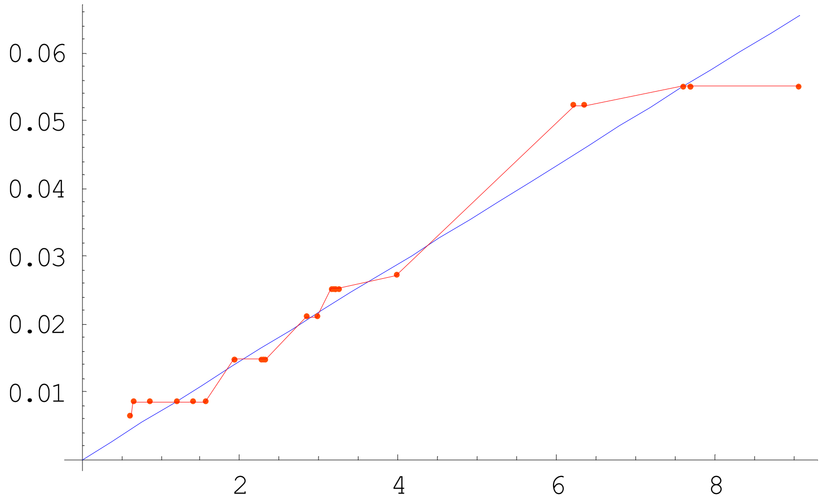

Figure 2

In Figure 3, carried out with the double majority rule (at least 13 countries with more than 60% of the population) proposed by the European Convention, we can appreciate the great increase in the Germans' power and the decrease in power of the Spaniards, Polish and all citizens of intermediate population's countries.

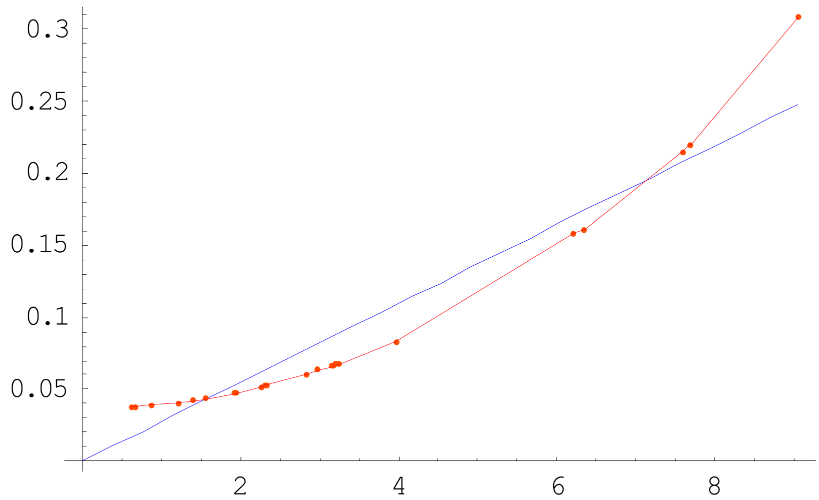

Figure 3

The inequality in the power of the citizens, with the rule which requires at least 13 countries with more than 66% of the population, increases in favour of the four big countries, as Figure 4 shows.

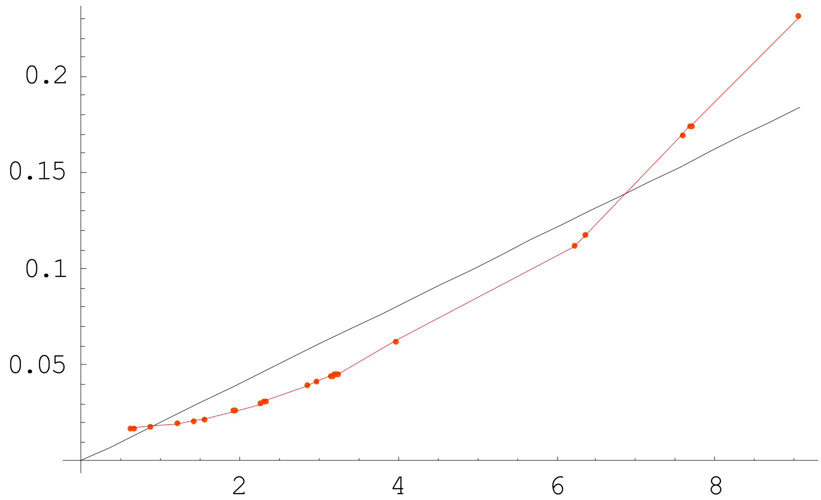

Figure 4

Acknowledgements

This research has been partially supported by the Spanish Ministry of Science and Technology, under grant SEC2003–00573.

References

[1] E. Algaba, J.M. Bilbao, J.R. Fernández and J.J. López. Computing power indices in weighted multiple majority games, *Mathematical Social Sciences* 46 (2003) 63–80.
[2] J.F. Banzhaf III. Weighted voting doesn't work: A mathematical analysis, *Rutgers Law Review* 19 (1965) 317–343.
[3] J.M. Bilbao, J.R. Fernández, N. Jiménez and J.J. López. Voting power in the European Union enlargement, *European Journal of Operational Research* 143 (2002) 181–196.
[4] T. Bräuninger and T. König. Indices of Power IOP 2.0 (2001), at `http://www.uni-konstanz.de/FuF/Verwiss/koenig/IOP.html`
[5] P. Dubey and L.S. Shapley. Mathematical properties of the Banzhaf power index, *Mathematics of Operations Research* 4 (1979) 99–131.
[6] D.S. Felsenthal and M. Machover. *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*, Edward Elgar, Cheltenham, 1998.
[7] D.S. Felsenthal and M. Machover. The Treaty of Nice and Qualified Majority Voting, *Social Choice and Welfare* 18 (2001) 431–464.
[8] L.S. Shapley and M. Shubik. A method for evaluating the distribution of power in a committee system, *American Political Science Review* 48 (1954) 787–792.

Department of Applied Mathematics II, Escuela Superior de Ingenieros, Camino de los Descubrimientos, 41092 Sevilla, Spain. E-mail: mbilbao@us.es